because method effects are now part of the residuals. Moreover, the CTUM model does not allow correlations between residuals of different methods. This might be necessary in the case of structurally different methods. Problems that are caused by trait-specific method effects can be appropriately handled in multiple indicator models.

Multiple Indicator Models

In multiple indicator models, there are several indicators for one trait–method unit. In the less restrictive model, there is one factor for all indicators belonging to the same trait–method unit. The correlations between these factors constitute a latent MTMM matrix. The correlation coefficients of this latent MTMM matrix are not distorted by measurement error and allow a more appropriate application of the Campbell and Fiske criteria for evaluating the MTMM matrix. Multiple indicator models allow the definition of trait-specific method factors and, therefore, the separation of measurement error and method-specific influences in a more appropriate way. Eid and colleagues have shown how different models of CFA can be defined for different types of methods. In the case of interchangeable methods, a multilevel CFA model can be applied that allows the specification of trait-specific method effects. In contrast to the extension of the CTCU model to multiple indicators, the multilevel approach has the advantage that the number of methods (e.g., raters) can differ between targets. In the case of structurally different raters, an extension of the CTC(M − 1) model to multiple indicators can be applied. This model allows a researcher to test specific hypotheses about the generalizability of method effects across traits and methods. In the case of a combination of structurally different and interchangeable methods, a multilevel CTC(M − 1) model would be appropriate.

Michael Eid

See also Construct Validity; "Convergent and Discriminant Validation by the Multitrait–Multimethod Matrix"; MBESS; Structural Equation Modeling; Triangulation; Validity of Measurement

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Multivalued Treatment Effects

The term *multivalued treatment effects* broadly refers to a collection of population parameters that capture the impact of a given treatment assigned to each observational unit, when this treatment status takes multiple values. In general, treatment levels may be finite or infinite as well as ordinal or cardinal, leading to a large collection of possible treatment effects to be studied in applications. When the treatment effect of interest is the mean

outcome for each treatment level, the resulting population parameter is typically called the *dose–response function* in the statistical literature, regardless of whether the treatment levels are finite or infinite. The analysis of multivalued treatment effects has several distinct features when compared with the analysis of binary treatment effects, including the following: (a) A comparison or control group is not always clearly defined, (b) new parameters of interest arise capturing distinct phenomena such as nonlinearities or tipping points, (c) in most cases correct statistical inferences require the joint estimation of all treatment effects (as opposed to the estimation of each treatment effect at a time), and (d) efficiency gains in statistical inferences may be obtained by exploiting known restrictions among the multivalued treatment effects. This entry discusses the treatment effect model and statistical inference procedures for multivalued treatment effects.

Treatment Effect Model and Population Parameters

A general statistical treatment effect model with multivalued treatment assignments is easily described in the context of the classical potential outcomes model. This model assumes that each unit i in a population has an underlying collection of potential outcome random variables $\{Y_i = Y_i(t) : t \in \mathcal{T}\}$, where $\mathcal{T}$ denotes the collection of possible treatment assignments. The random variables $Y_i(t)$ are usually called *potential outcomes* because they represent the random outcome that unit i would have under treatment regime $t \in \mathcal{T}$. For each unit i and for any two treatment levels, t_1 and t_2, it is always possible to define the *individual treatment effect* given by $Y_i(t_1) - Y_i(t_2)$, which may or may not be a degenerate random variable. However, because units are not observed under different treatment regimes simultaneously, such comparisons are not feasible. This idea, known as the *fundamental problem of causal inference,* is formalized in the model by assuming that for each unit i only (Y_i, T_i) is observed, where $Y_i = Y_i(T_i)$ and $T_i \in \mathcal{T}$. In words, for each unit i, only the potential outcome for treatment level $T_i = t$ is observed while all other (counterfactual) outcomes are missing. Of course, in most applications, which treatment each unit has taken up is not random and hence further assumptions would be needed to identify the treatment effect of interest.

A binary treatment effect model has $\mathcal{T} = \{0, 1\}$, a finite multivalued treatment effect model has $\mathcal{T} = \{0, 1, \ldots, J\}$ for some positive integer J, and a continuous treatment effect model has $\mathcal{T} = [0, 1]$. (Note that the values in $\mathcal{T}$ are ordinal, that is, they may be seen just as normalizations of the underlying real treatment levels in a given application.) Many applications focus on a binary treatment effects model and base the analysis on the comparison of two groups, usually called *treatment group* $(T_i = 1)$ and *control group* $(T_i = 0)$. A multivalued treatment may be collapsed into a binary treatment, but this procedure usually would imply some important loss of information in the analysis. Important phenomena such as nonlinearities, differential effects across treatment levels or tipping points, cannot be captured by a binary treatment effect model.

Typical examples of multivalued treatment effects are comparisons between some characteristic of the distributions of the potential outcomes. Well-known examples are mean and quantile comparisons, although in many applications other features of these distributions may be of interest. For example, assuming, to simplify the discussion, that the random potential outcomes are equal for all units (this holds, for instance, in the context of random sampling), the mean of the potential outcome under treatment regime $t \in \mathcal{T}$ is given by $\mu(t) = \mathrm{E}[Y_i(t)]$. The collection of these means is the so-called dose–response function. Using this estimand, it is possible to construct different multivalued treatment effects of interest, such as pairwise comparisons (e.g., $\mu(t_2) - \mu(t_1)$) or differences in pairwise comparisons, which would capture the idea of nonlinear treatment effects. (In the particular case of binary treatment effects, the only possible pairwise comparison is $\mu(1) - \mu(0)$, which is called the *average treatment effect.*) Using the dose–response function, it is also possible to consider other treatment effects that arise as nonlinear transformations of $\mu(t)$, such as ratios, incremental changes, tipping points, or the maximal treatment effect $\mu^* = max_{t \in \mathcal{T}} \mu(t)$, among many other possibilities. All these multivalued treatment effects are constructed on the basis of

the mean of the potential outcomes, but similar estimands may be considered that are based on quantiles, dispersion measures, or other characteristics of the underlying potential outcome distribution. Conducting valid hypothesis testing about these treatment effects requires in most cases the joint estimation of the underlying multivalued treatment effects.

Statistical Inference

There exists a vast theoretical literature proposing and analyzing different statistical inference procedures for multivalued treatment effects. This large literature may be characterized in terms of the key identifying assumption underlying the treatment effect model. This key assumption usually takes the form of a (local) independence or orthogonality condition, such as (a) a conditional independence assumption, which assumes that conditional on a set of observable characteristics, selection into treatment is random, or (b) an instrumental variables assumption, which assumes the existence of variables that induce exogenous changes in the treatment assignment. With the use of an identifying assumption (together with other standard model assumptions), it has been shown in the statistical and econometrics literatures that several parametric, semiparametric, and nonparametric procedures allow for optimal joint inference in the context of multivalued treatments. These results are typically obtained with the use of large sample theory and justify (asymptotically) the use of classical statistical inference procedures involving multiple treatment levels.

Matias D. Cattaneo

See also Multiple Treatment Interference; Observational Research; Propensity Score Analysis; Selection; Treatment(s)

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Multivariate Analysis of Variance (MANOVA)

Multivariate analysis of variance (MANOVA) designs are appropriate when multiple dependent variables are included in the analysis. The dependent variables should represent continuous measures (i.e., interval or ratio data). Dependent variables should be moderately correlated. If there is no correlation at all, MANOVA offers no improvement over an analysis of variance (ANOVA); if the variables are highly correlated, the same variable may be measured more than once. In many MANOVA situations, multiple independent variables, called factors, with multiple levels are included. The independent variables should be categorical (qualitative). Unlike ANOVA procedures that analyze differences across two or more groups on one dependent variable, MANOVA procedures analyze differences across two or more groups on two or more dependent variables. Investigating two or more dependent variables simultaneously is important in various disciplines, ranging from the natural and physical sciences to government and business and to the behavioral and social sciences. Many research questions cannot be answered adequately by an investigation of only one dependent variable because treatments in experimental studies are likely to affect subjects in more than one way. The focus of this entry is on the various types of MANOVA procedures and associated assumptions. The logic of MANOVA and advantages and disadvantages of MANOVA are included.

MANOVA is a special case of the general *linear models.* MANOVA may be represented in a basic linear equation as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, where $\mathbf{Y}$ represents a vector of dependent variables, $\mathbf{X}$ represents