

Supplemental Material to “Robust Data-Driven Inference for Density-Weighted  
Average Derivatives”\*

MATIAS D. CATTANEO

DEPARTMENT OF ECONOMICS, UNIVERSITY OF MICHIGAN

RICHARD K. CRUMP

FEDERAL RESERVE BANK OF NEW YORK

MICHAEL JANSSON

DEPARTMENT OF ECONOMICS, UC BERKELEY AND *CREATES*

March 26, 2010

1. DESCRIPTION

This document contains a comprehensive set of results from the Monte Carlo experiment summarized in Section 5 of the paper entitled “Robust Data-Driven Inference for Density-Weighted Average Derivatives”. These results include all combinations of sample sizes ( $n = 100$ ,  $n = 400$ ,  $n = 700$ ), dimension of regressors vector ( $d = 2$ ,  $d = 4$ ), and kernel orders ( $P = 2$ ,  $P = 4$ ).

Figures 1 through 12 plot the empirical coverage for the three competing 95% confidence intervals as a function of the choice of bandwidth for each of the six models. Figures 13 through 24 plot kernel density estimates for the test statistic PSS coupled with either  $h_{PS}^*$  and  $h_{NR}^*$ , and for the test statistics CCJ1 and CCJ2 coupled with  $h_{CCJ}^*$ .

Tables 1 through 6 report empirical coverage of each possible confidence intervals (PSS, CCJ1, CCJ2) when using each possible population bandwidth selector ( $h_{PS}^*$ ,  $h_{NR}^*$ ,  $h_{CCJ}^*$ ). Tables 7 through 12 report average empirical bias and average empirical interval length for each

---

\*The first author gratefully acknowledges financial support from the National Science Foundation (SES 0921505). The third author gratefully acknowledges financial support from the National Science Foundation (SES 0920953) and the research support of CREATES (funded by the Danish National Research Foundation).

competing confidence interval when coupled with each possible population bandwidth selector. Tables 13 through 18 report empirical coverage of each possible confidence interval (PSS, CCJ1, CCJ2) when using each possible estimated bandwidth selector  $(\hat{h}_{PS}, \hat{h}_{NR}, \hat{h}_{CCJ})$ .

Further results, and computer code in **R** with **C** implementations, are available upon request.

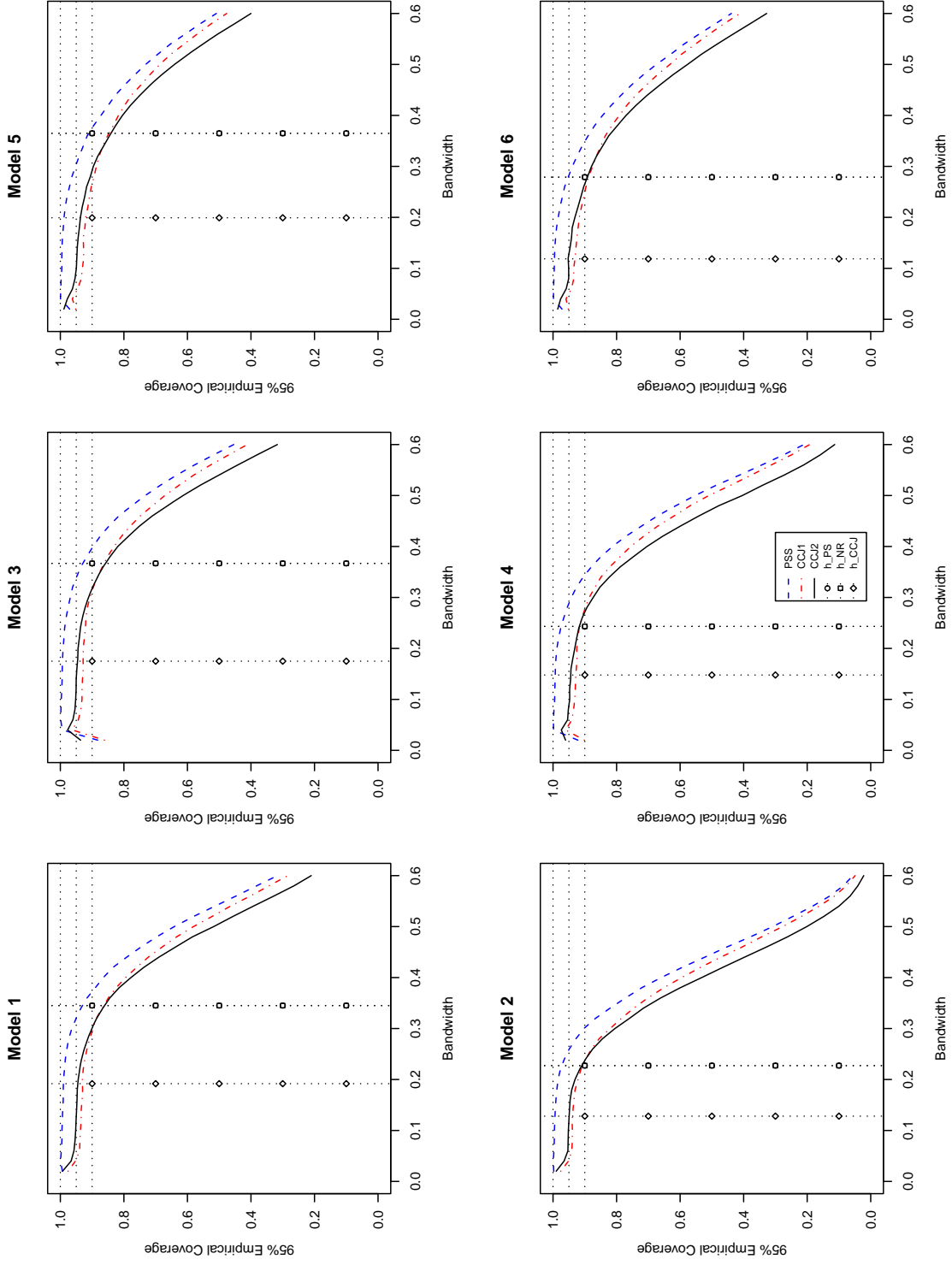


Figure 1: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 2$ ,  $n = 100$

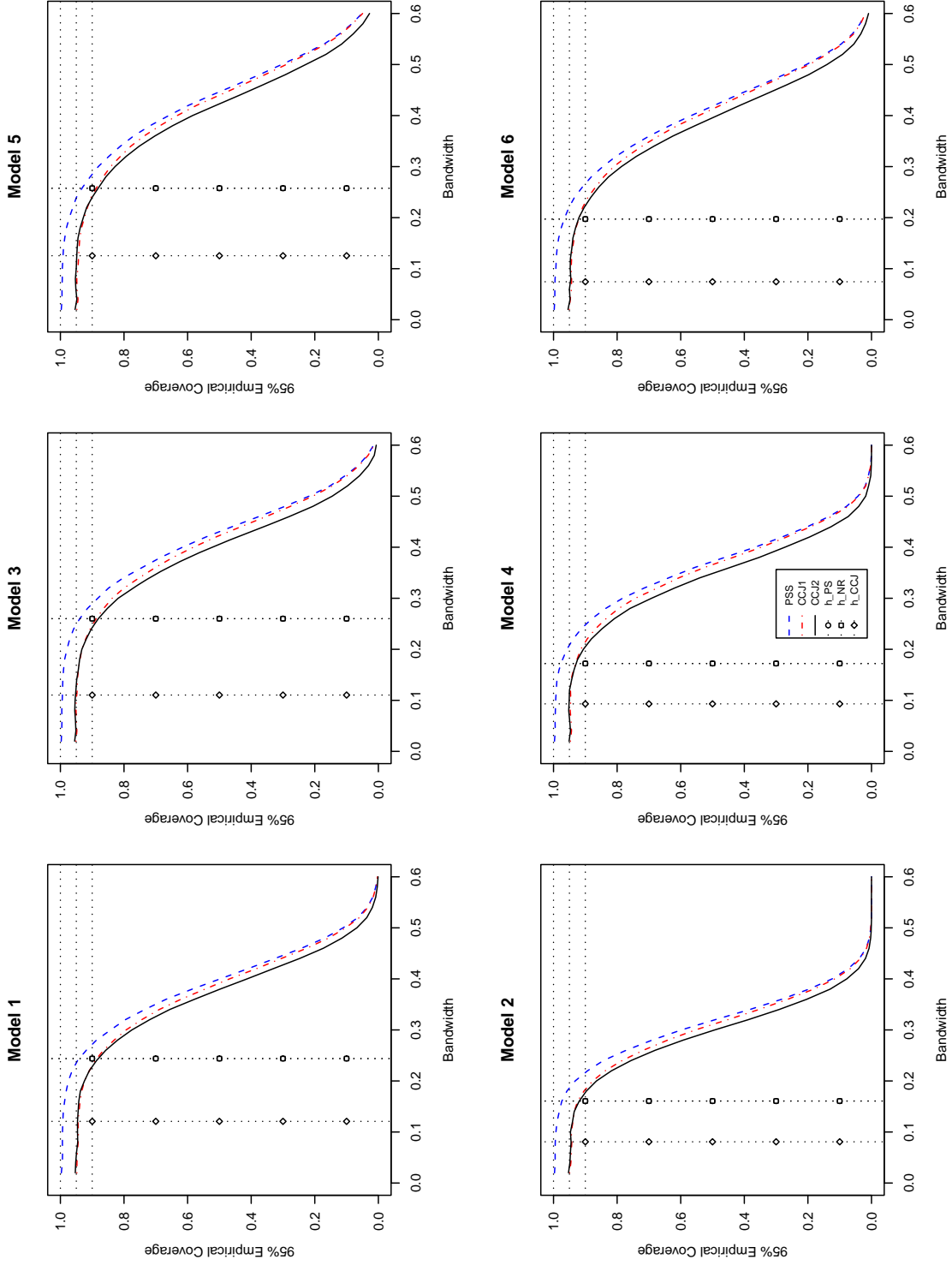


Figure 2: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2, P = 2, n = 400$

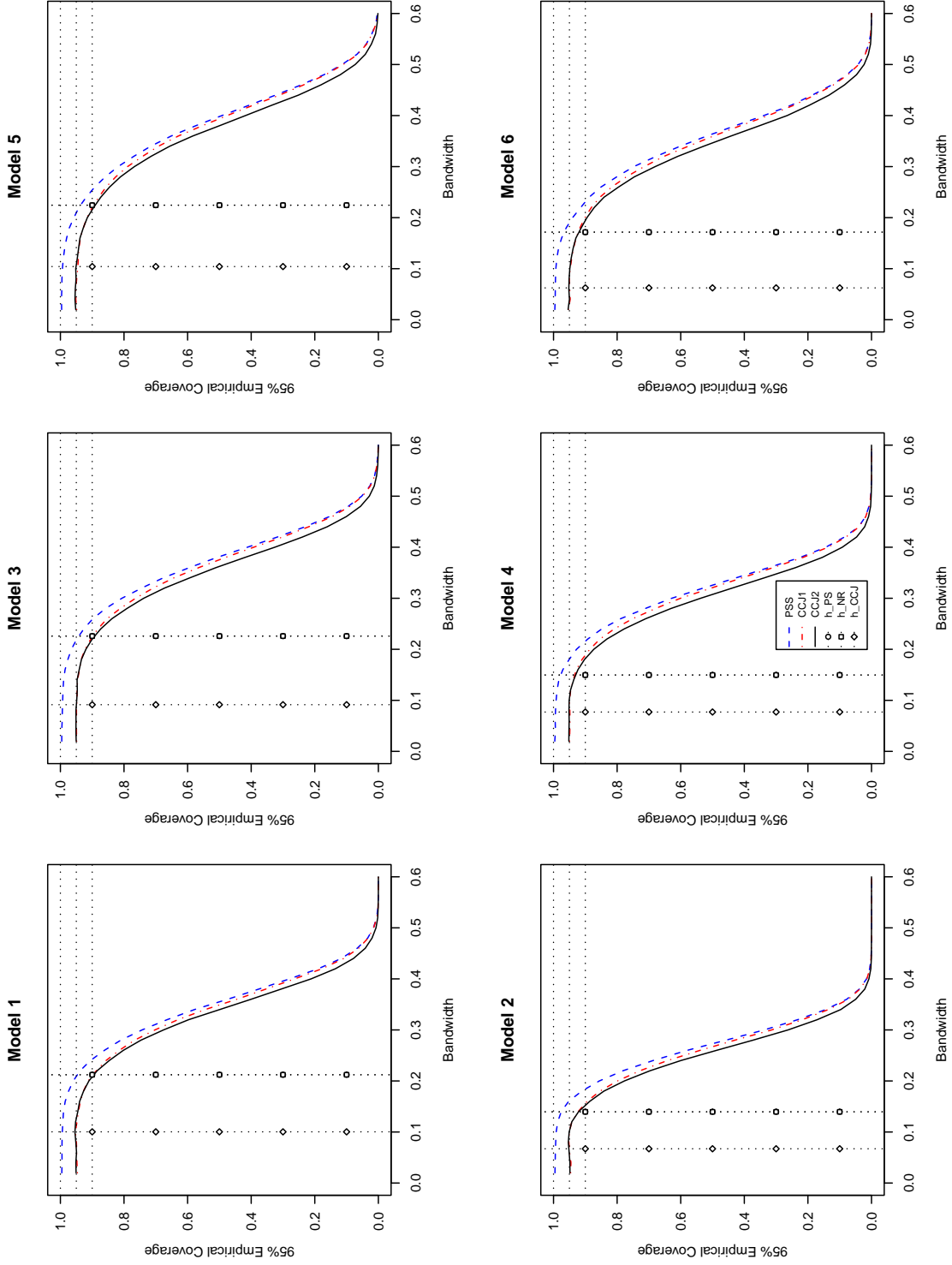


Figure 3: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 2$ ,  $n = 700$

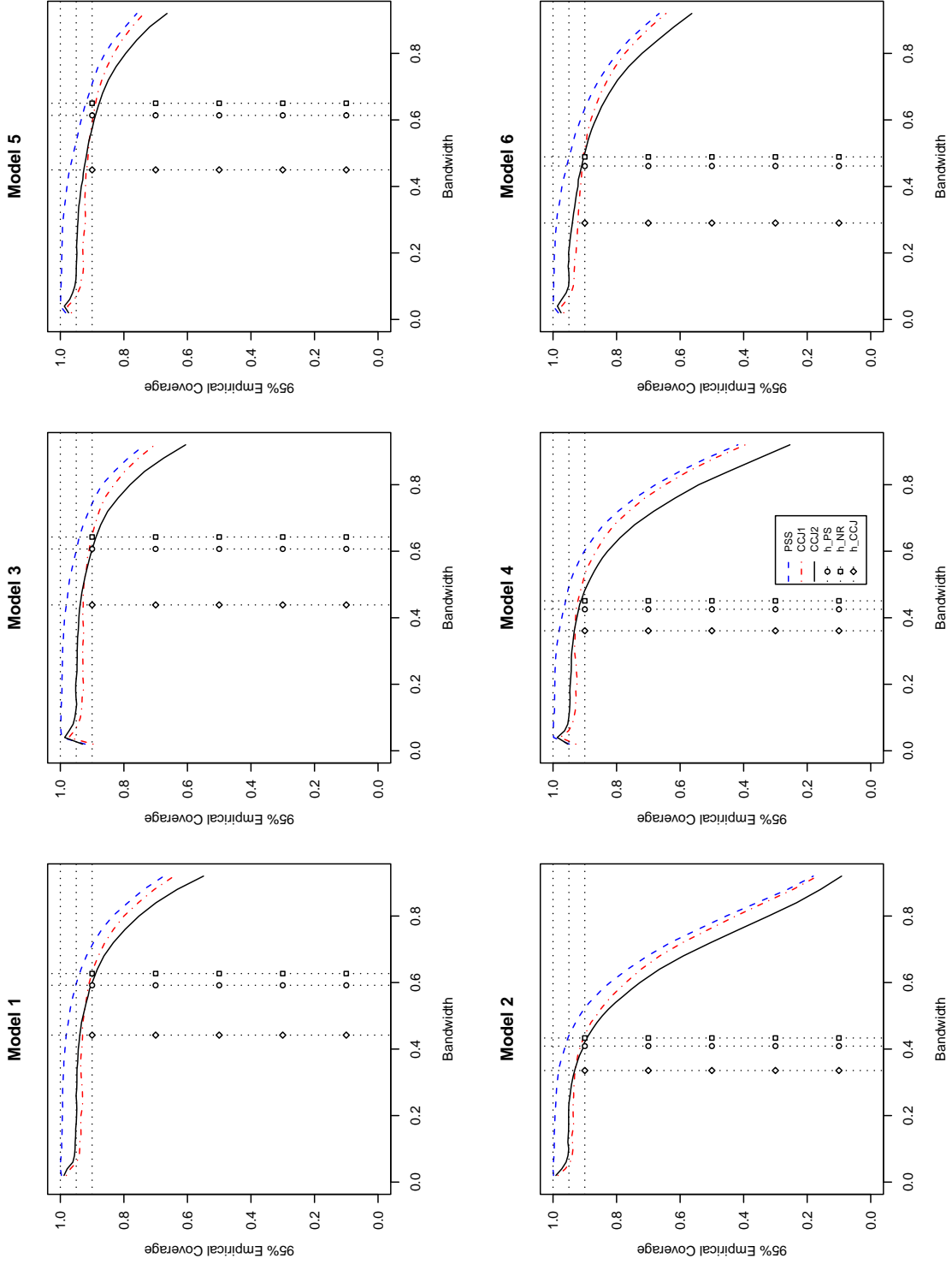


Figure 4: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 4$ ,  $n = 100$

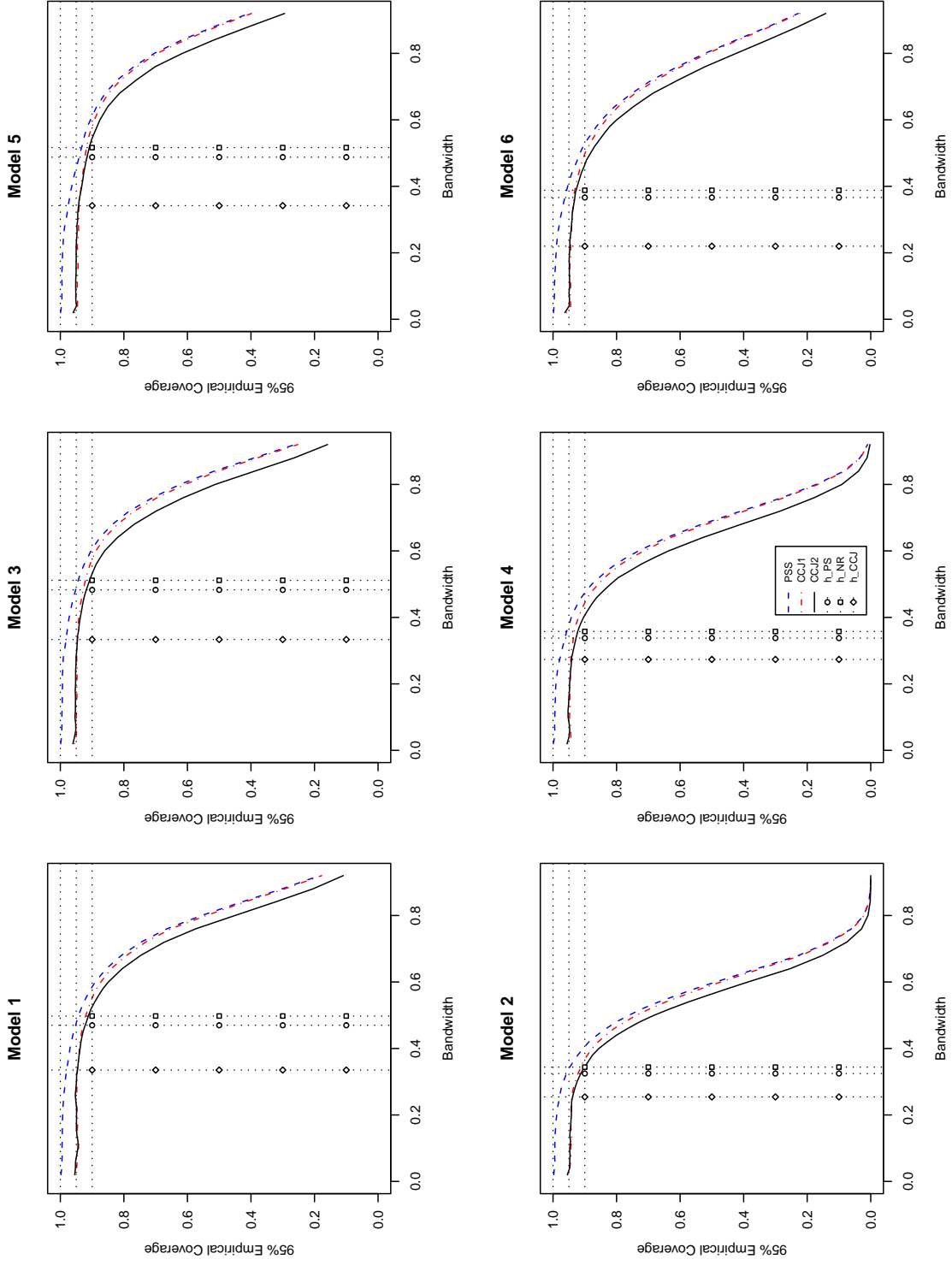


Figure 5: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2, P = 4, n = 400$

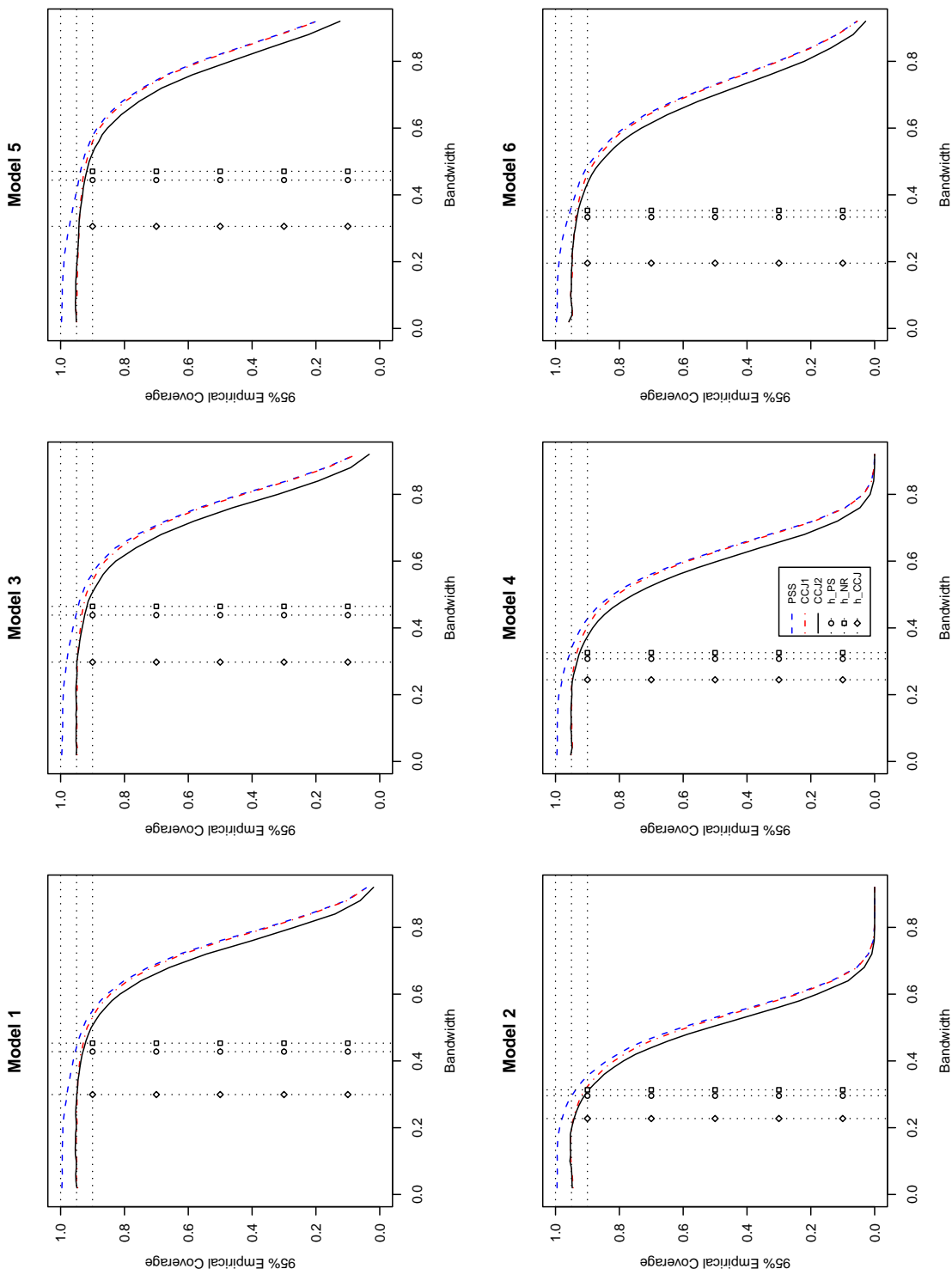


Figure 6: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 4$ ,  $n = 700$



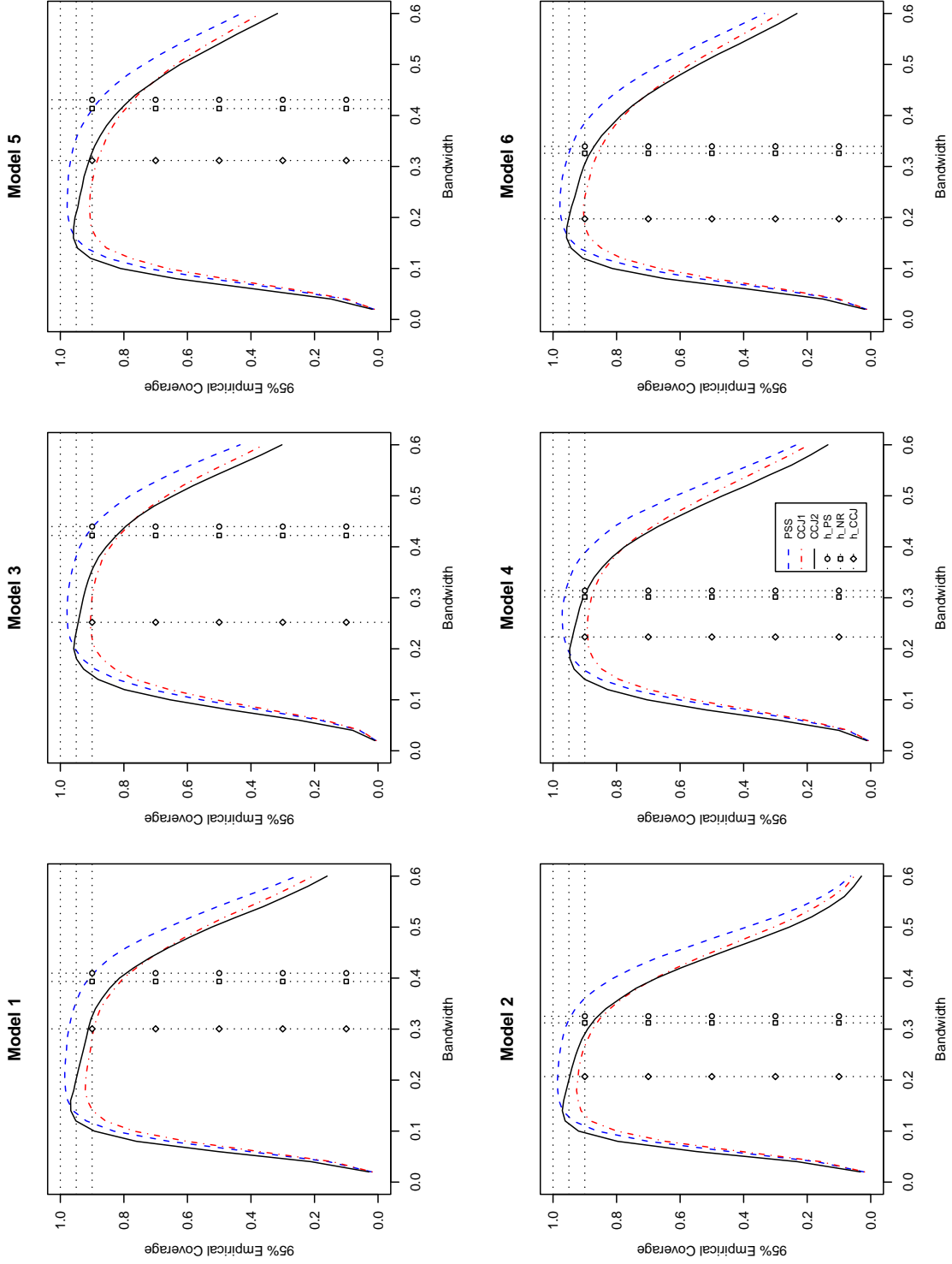


Figure 7: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4, P = 2, n = 100$

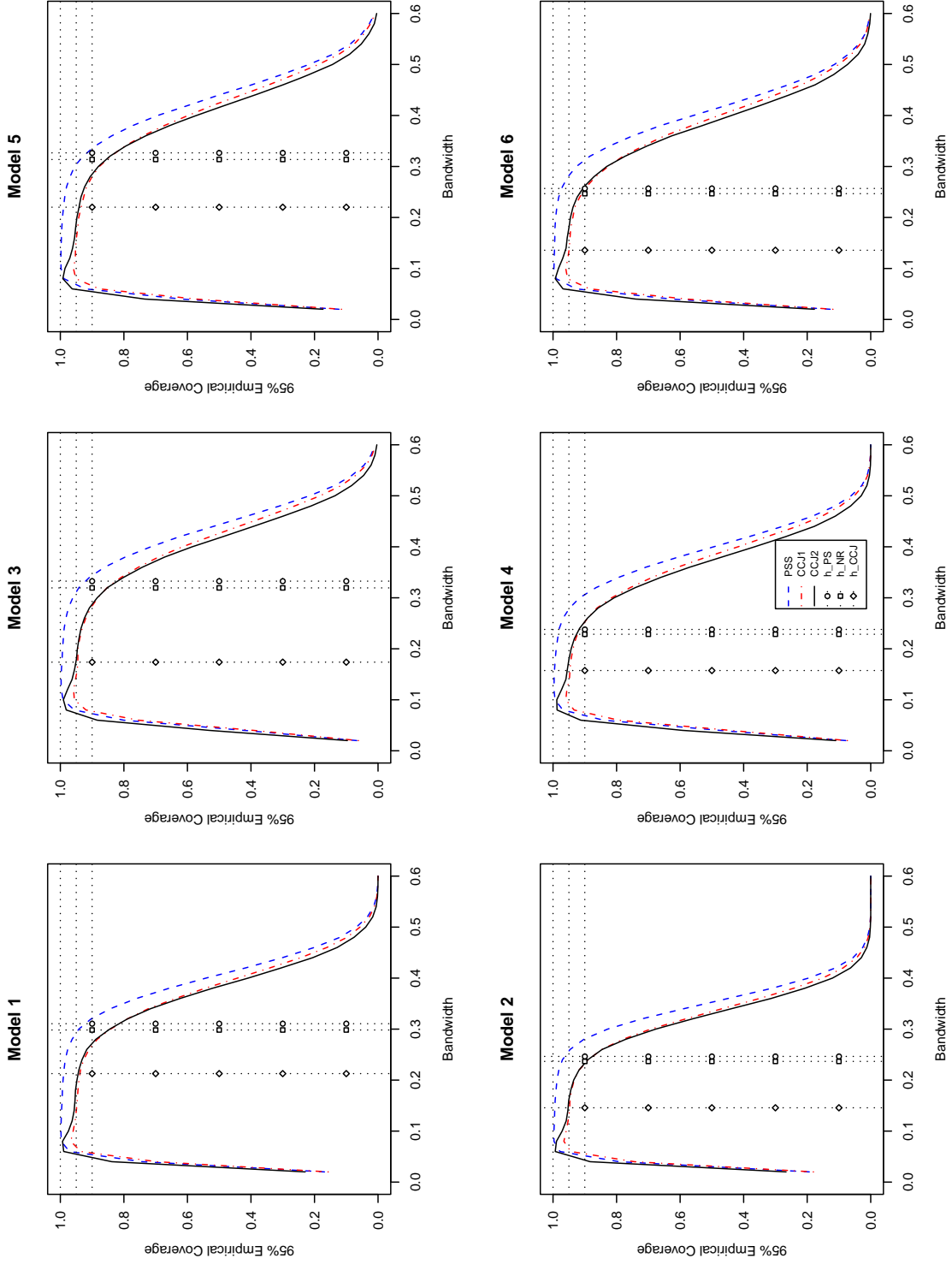


Figure 8: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4, P = 2, n = 400$

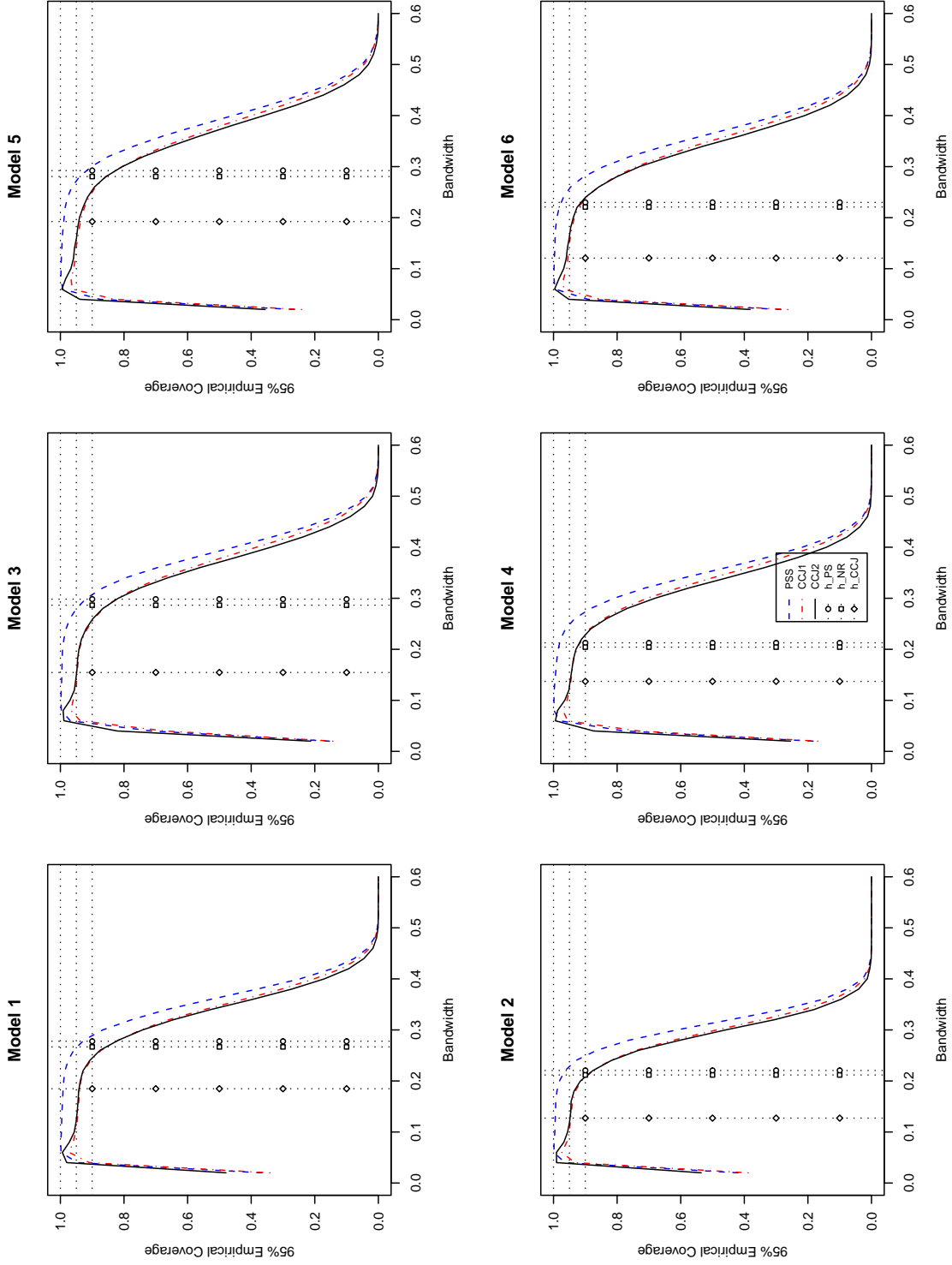


Figure 9: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4, P = 2, n = 700$

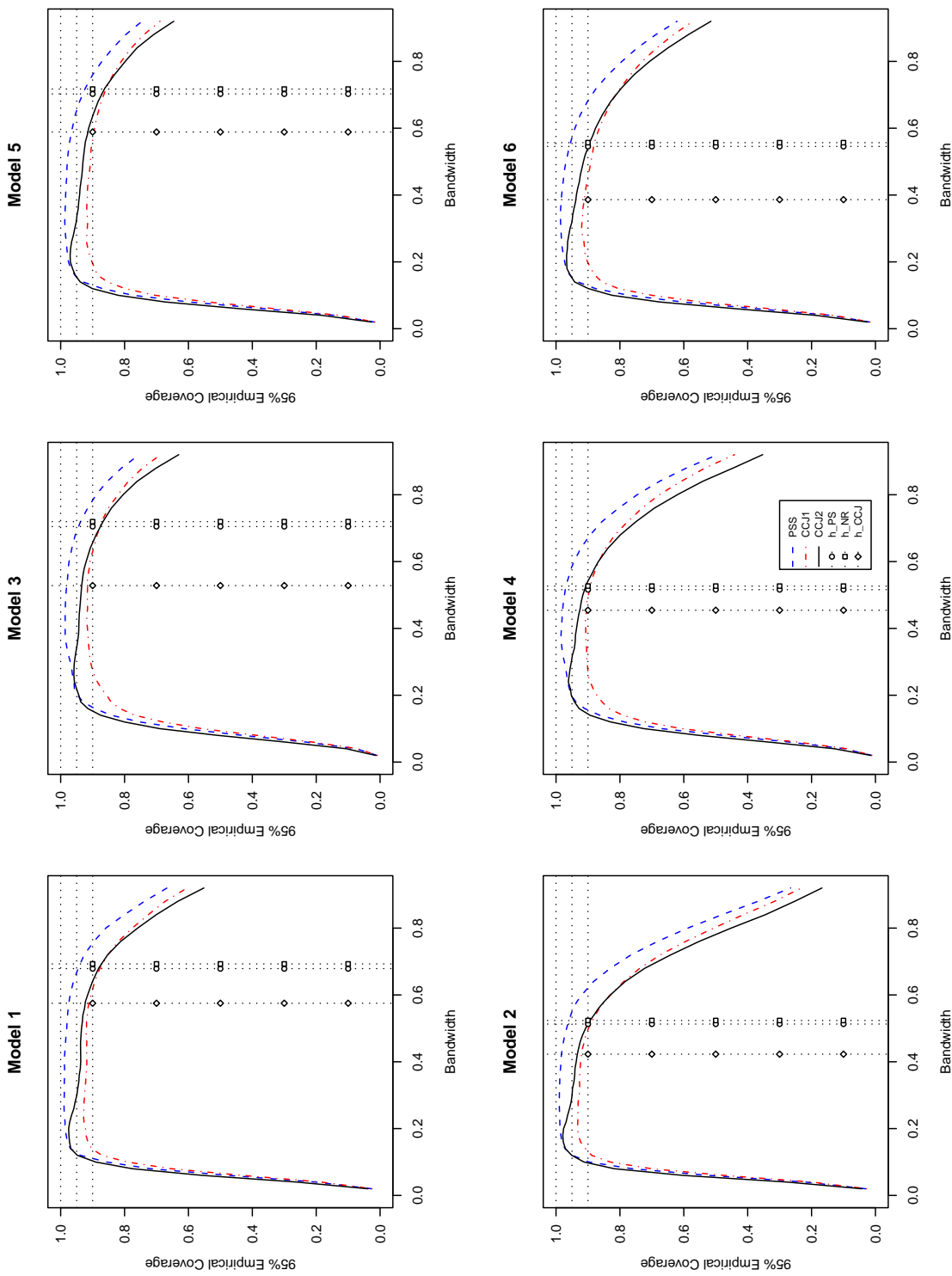


Figure 10: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 4$ ,  $n = 100$

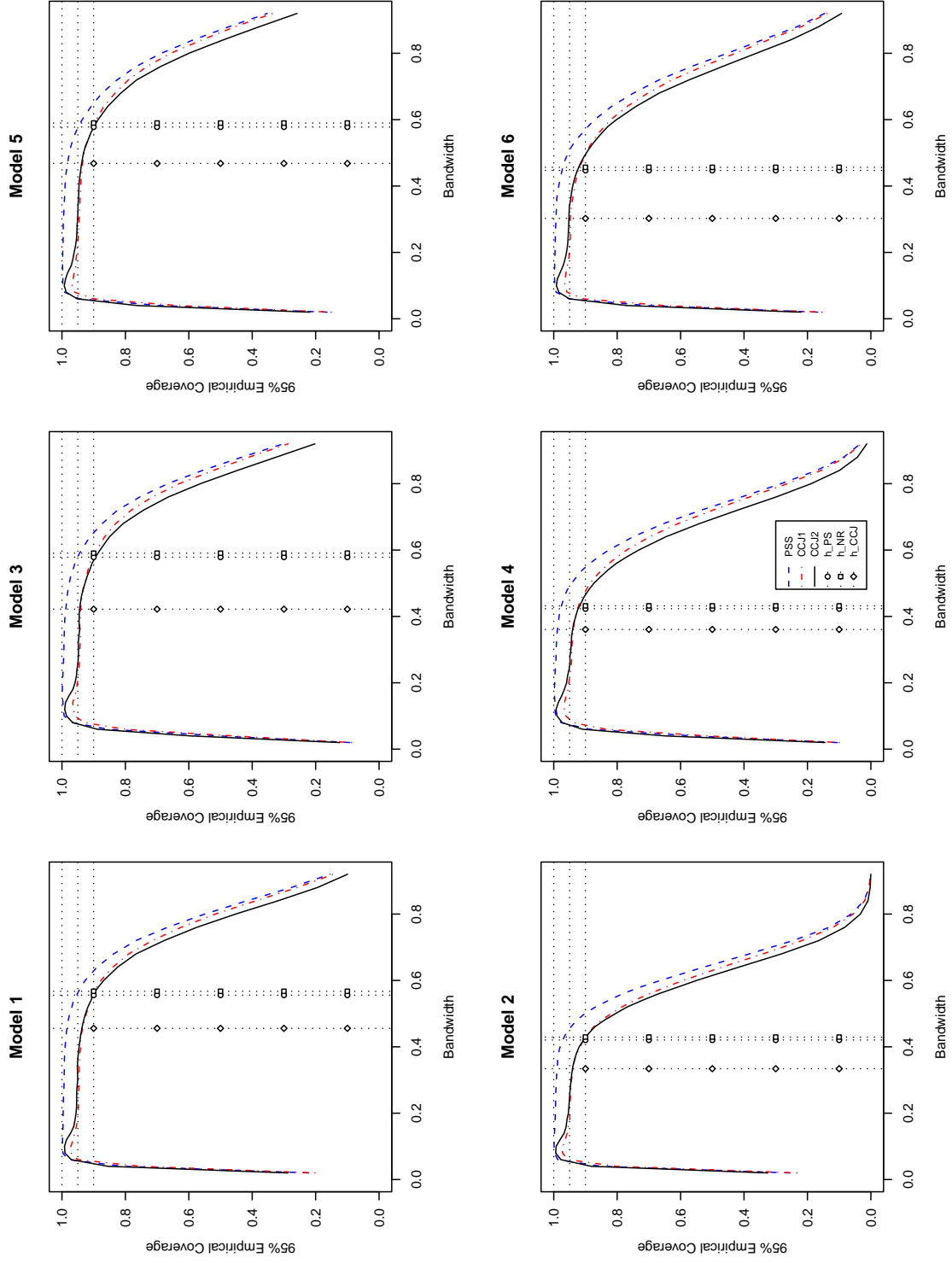


Figure 11: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 4$ ,  $n = 400$

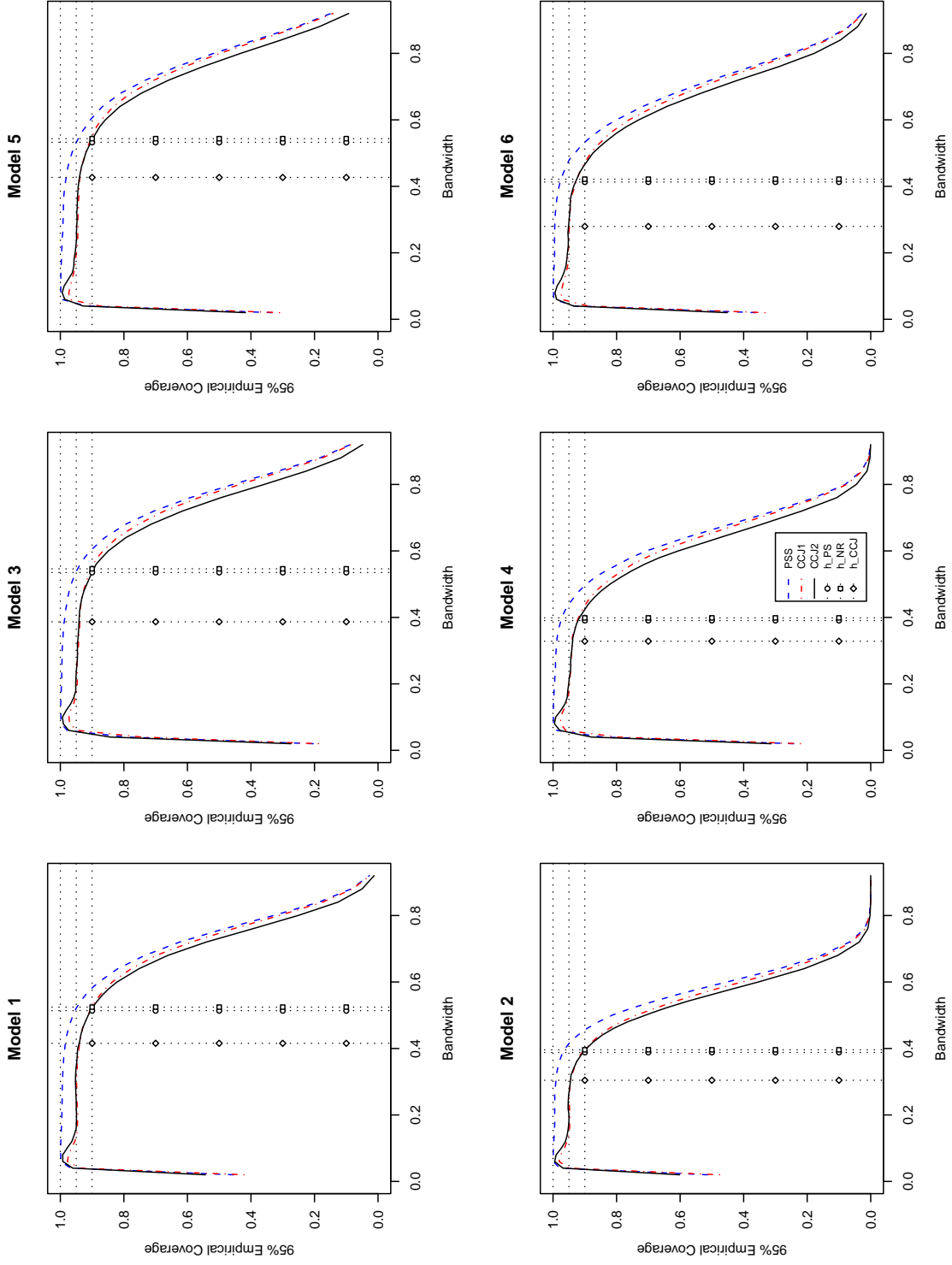


Figure 12: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 4$ ,  $n = 700$

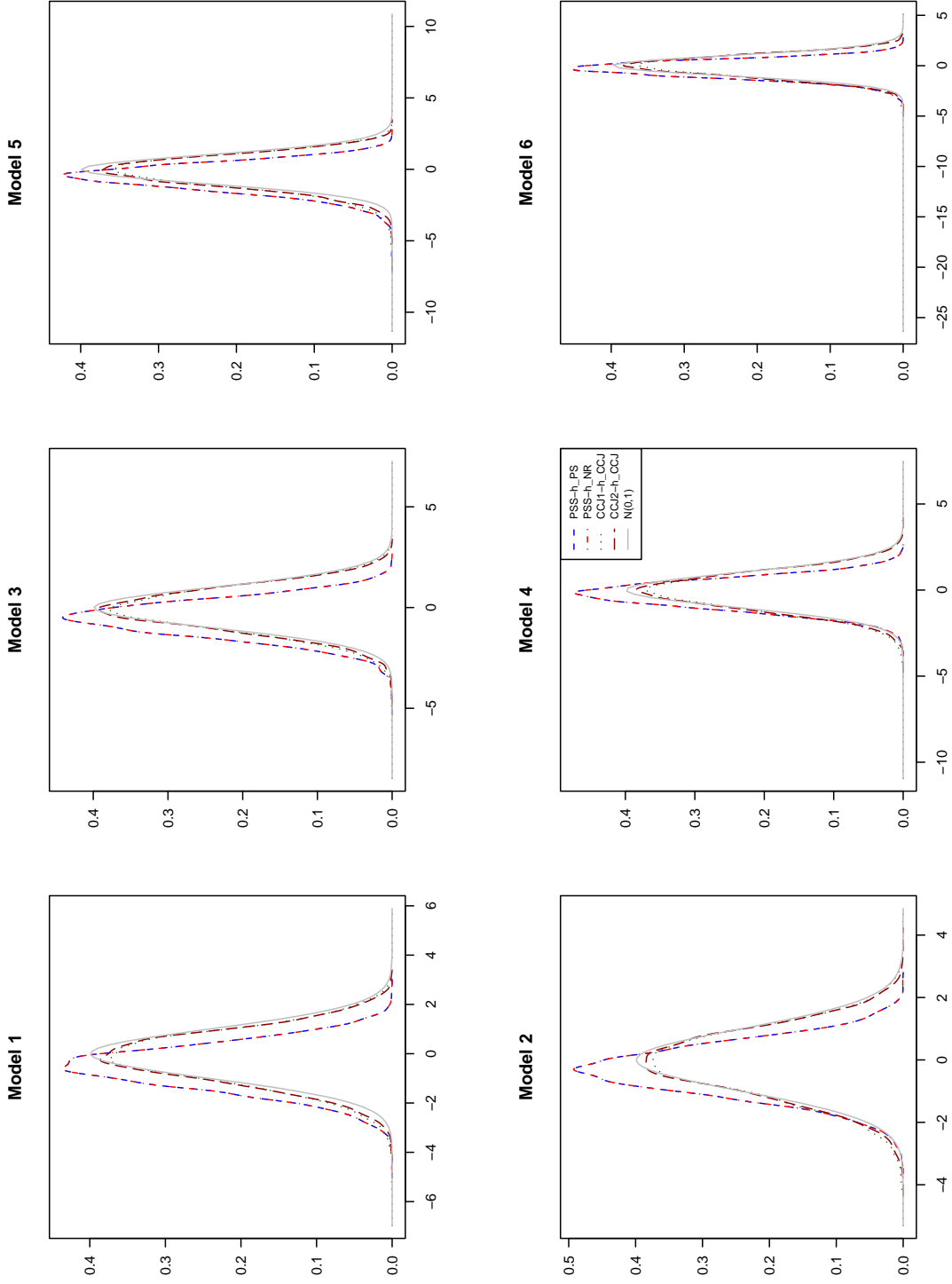


Figure 13: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 2$ ,  $n = 100$

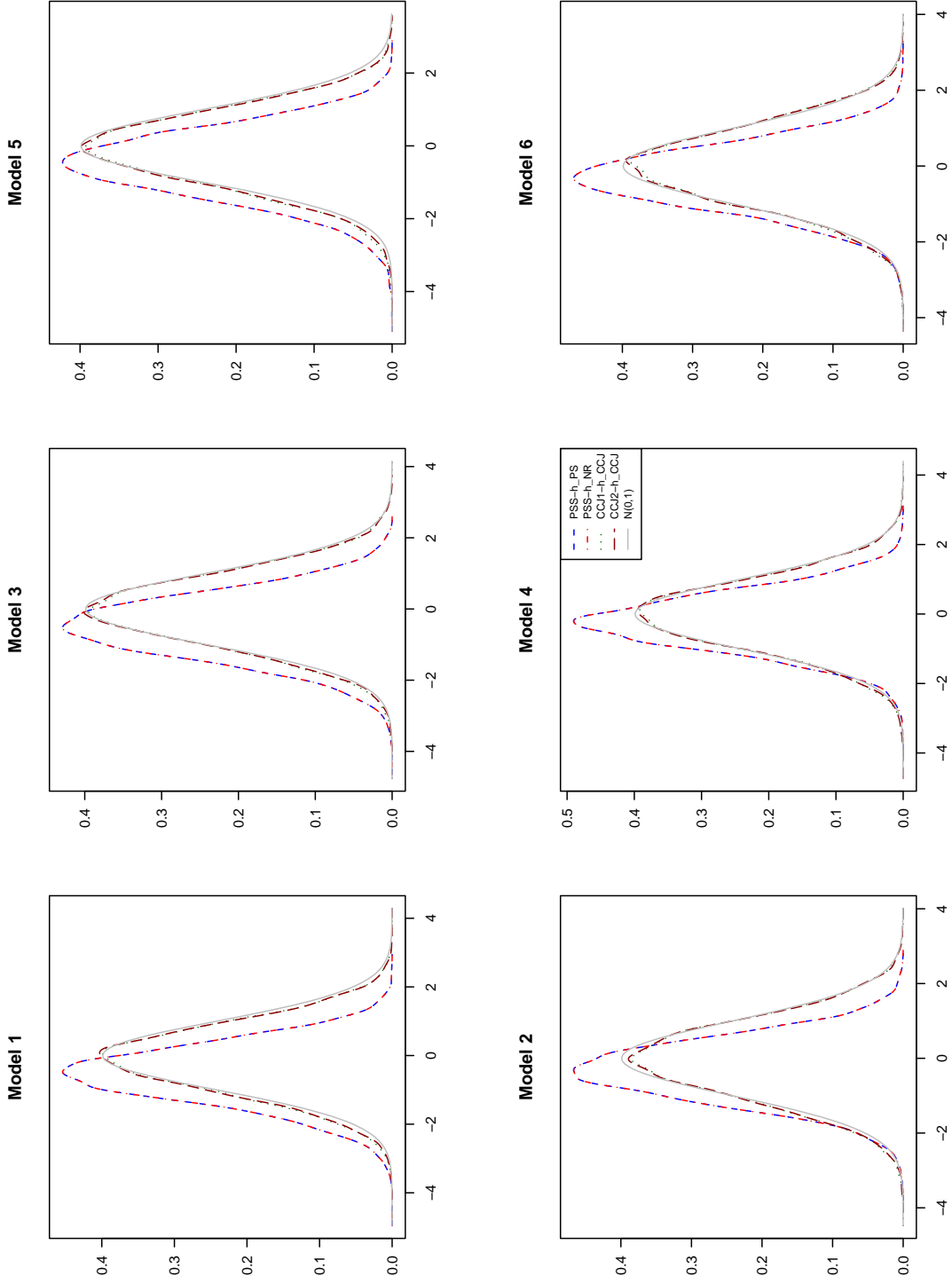


Figure 14: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 2$ ,  $n = 400$



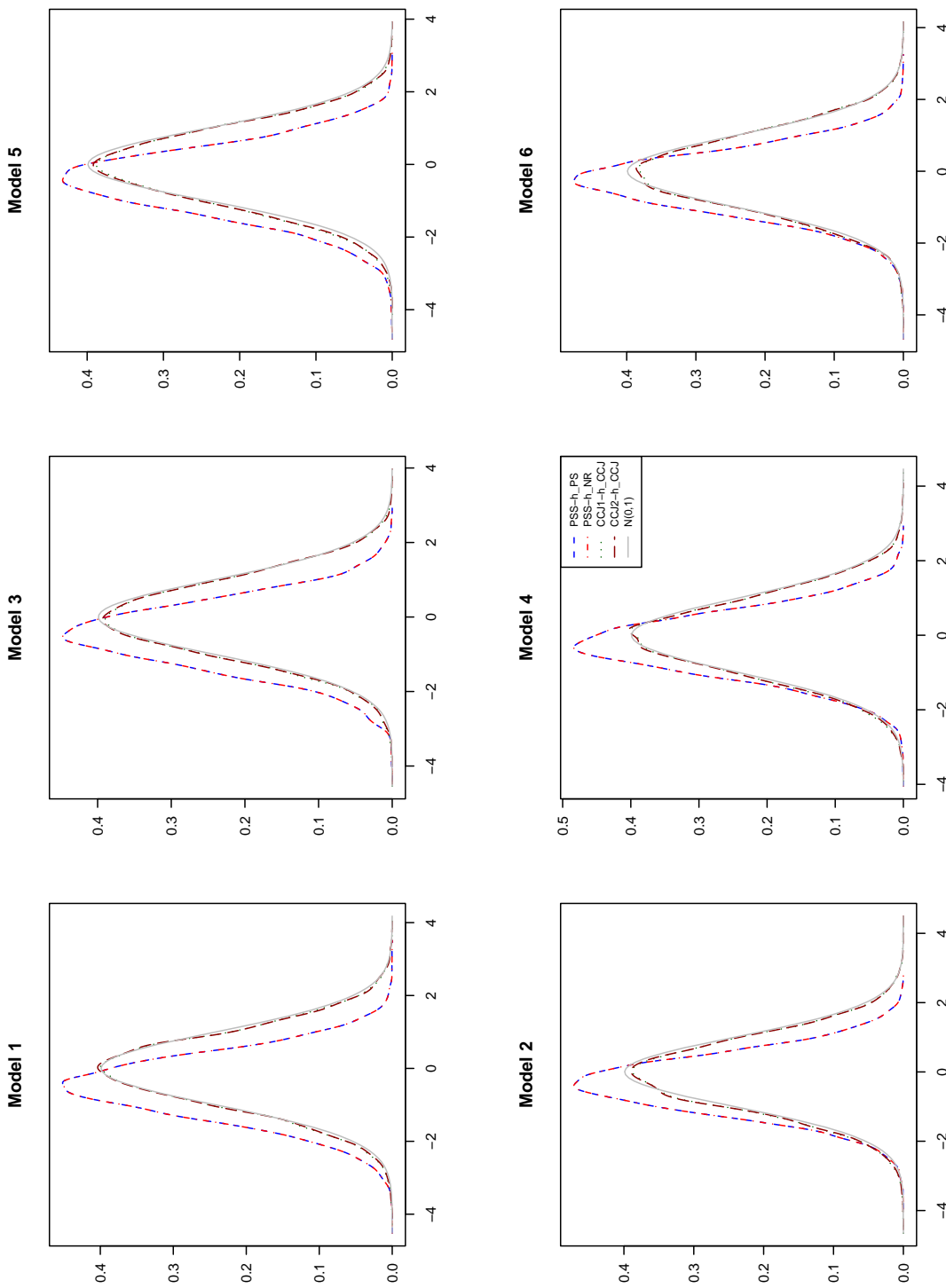


Figure 15: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2$ ,  $P = 2$ ,  $n = 700$

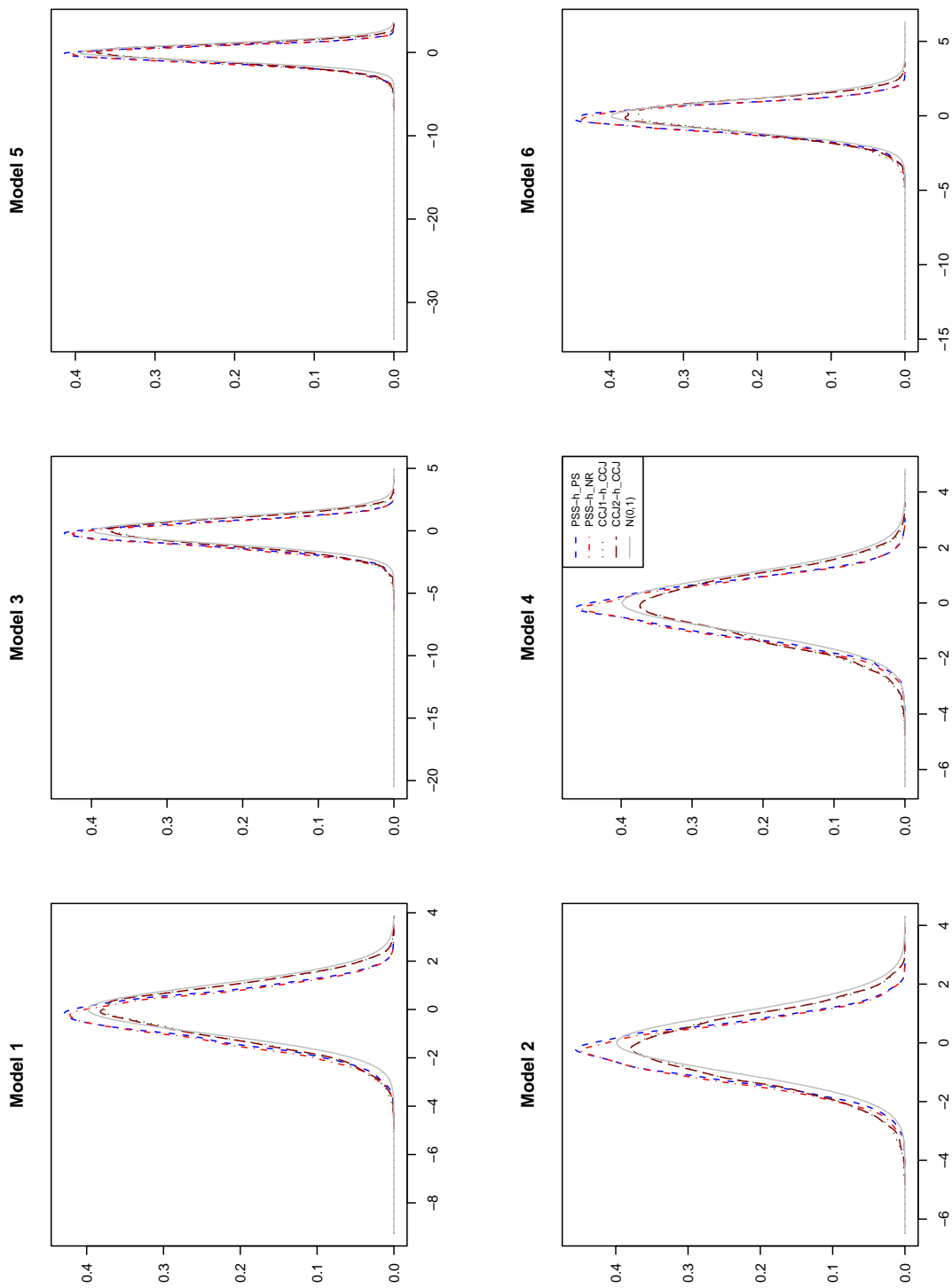


Figure 16: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2, P = 4, n = 100$

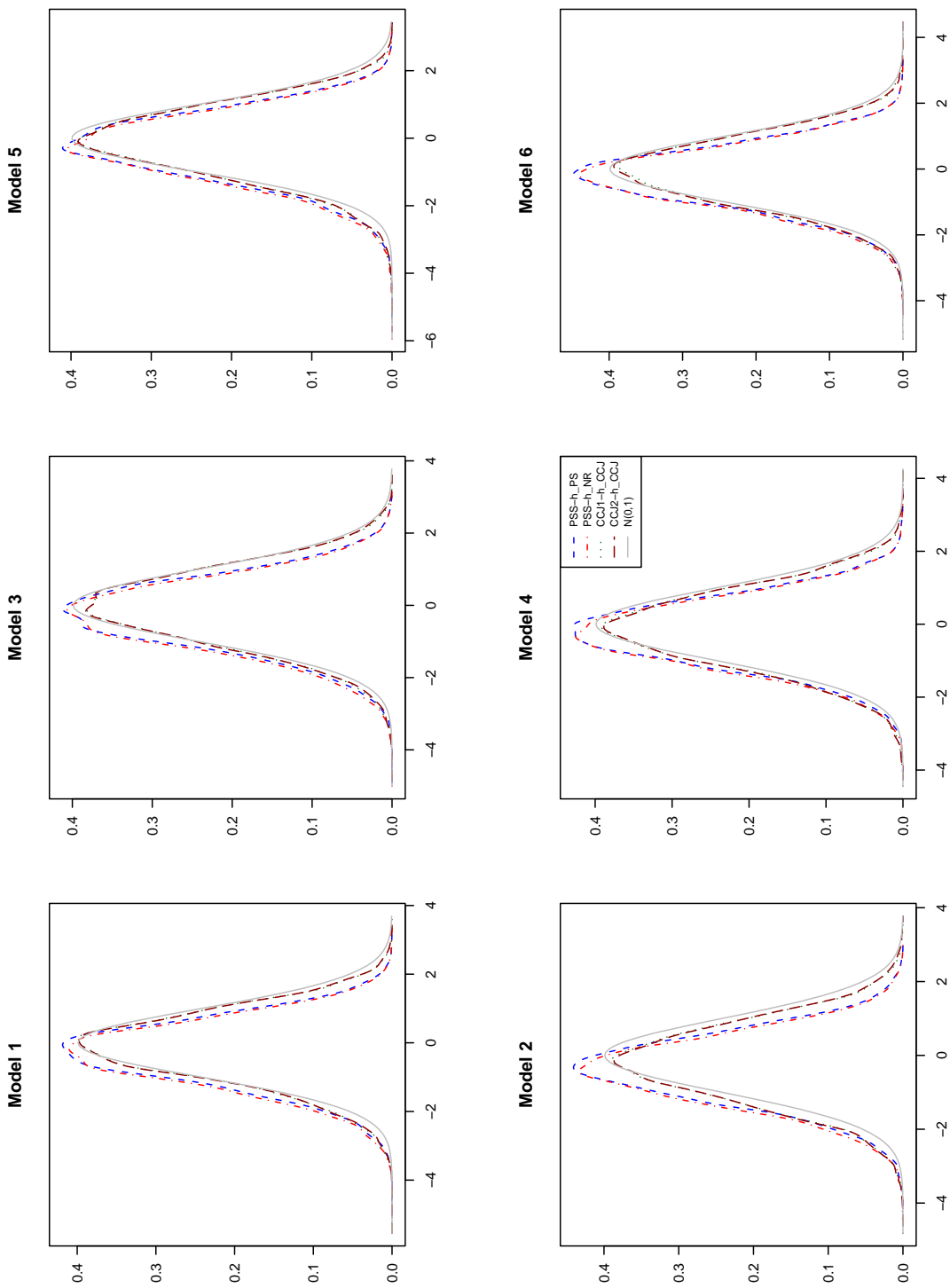


Figure 17: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2, P = 4, n = 400$

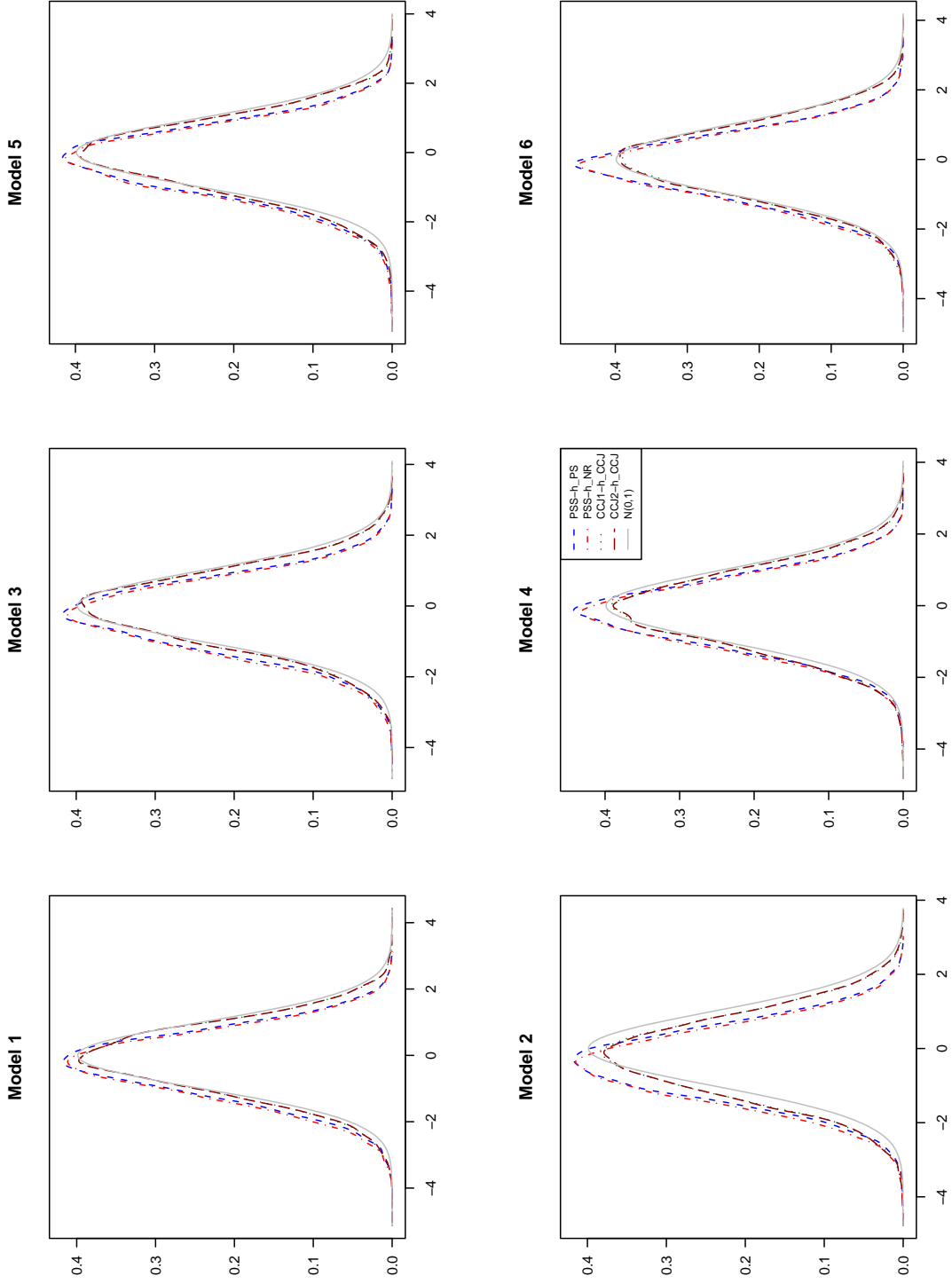


Figure 18: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 2, P = 4, n = 700$

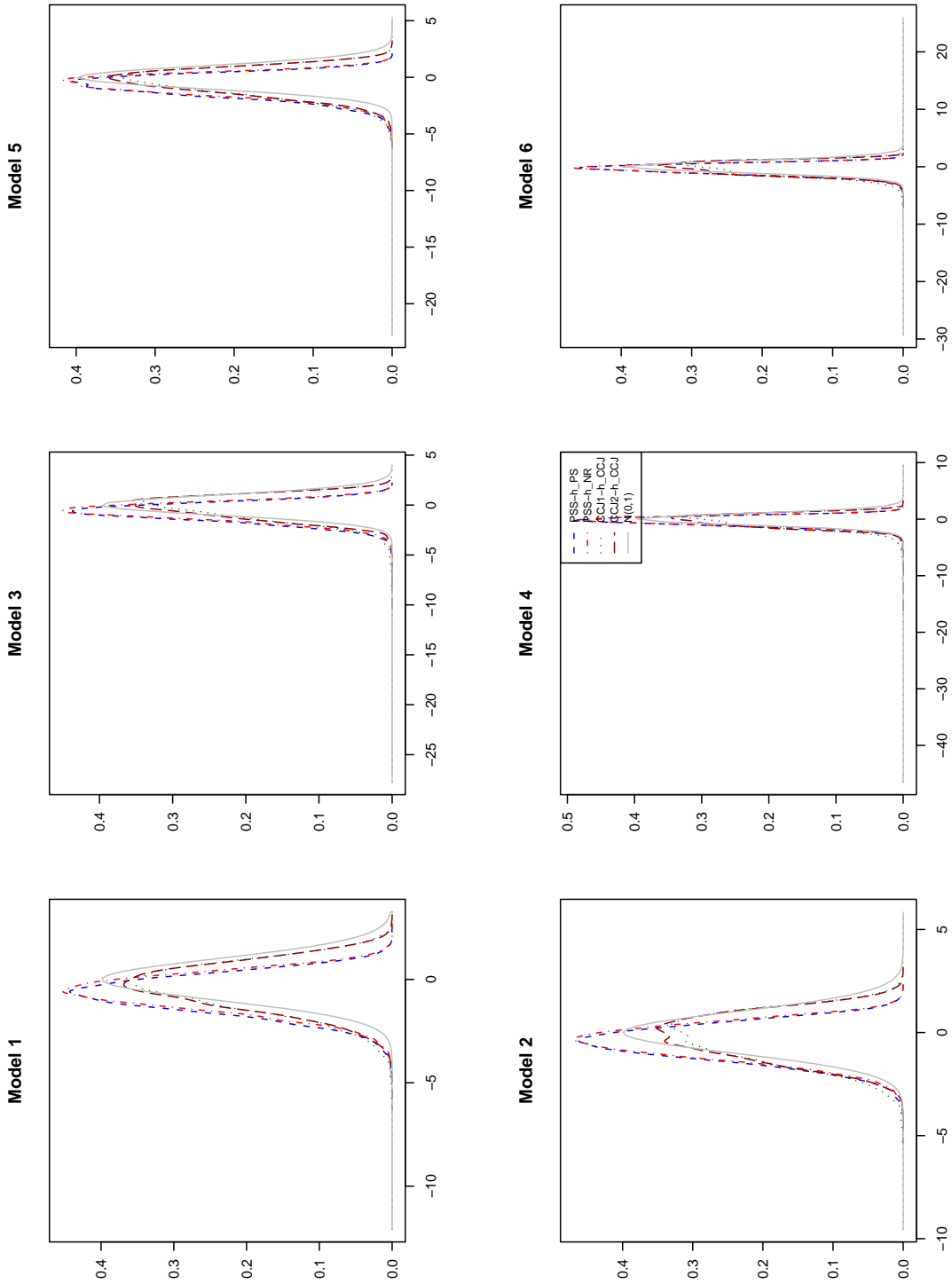


Figure 19: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 2$ ,  $n = 100$

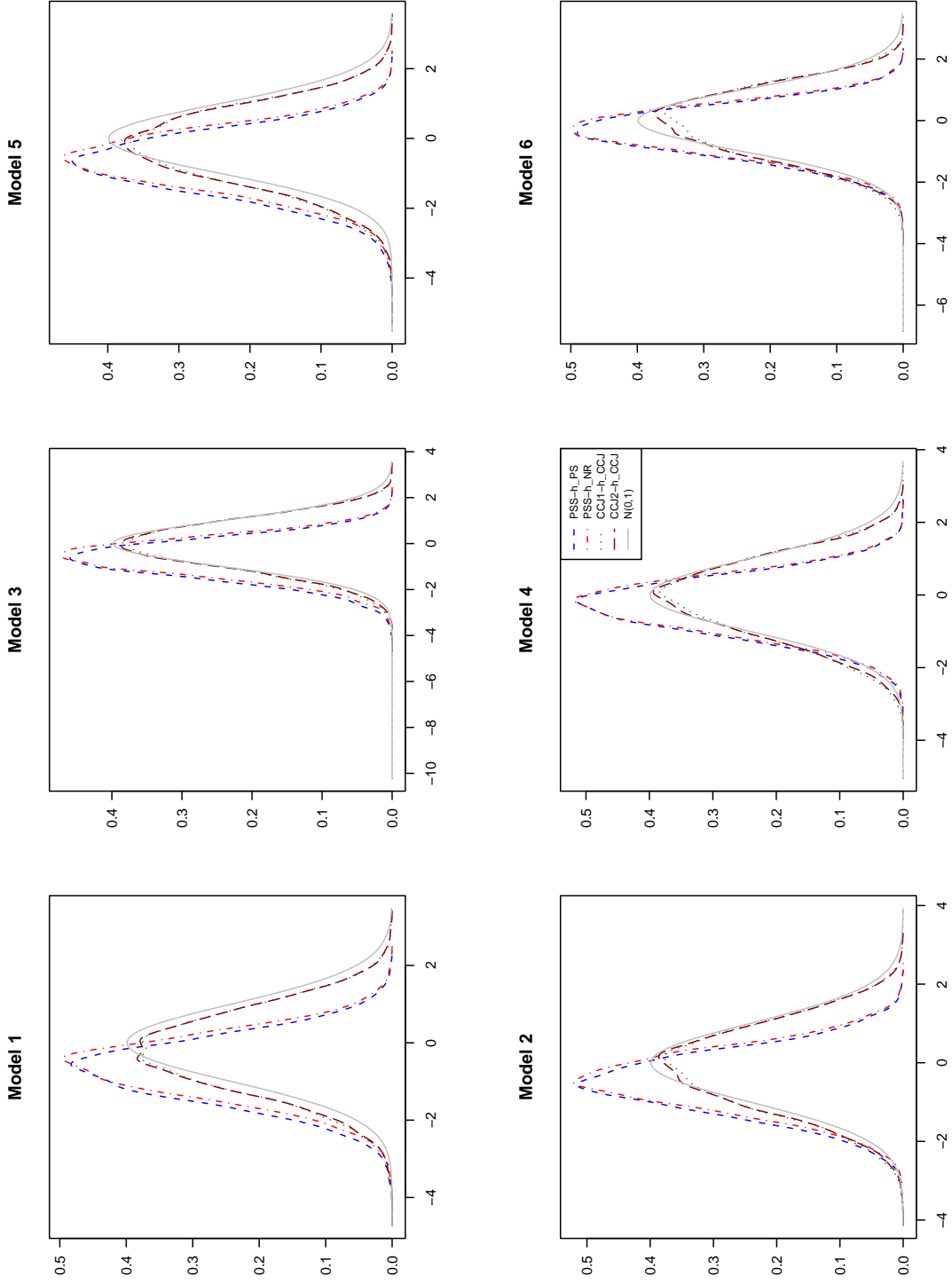


Figure 20: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 2$ ,  $n = 400$

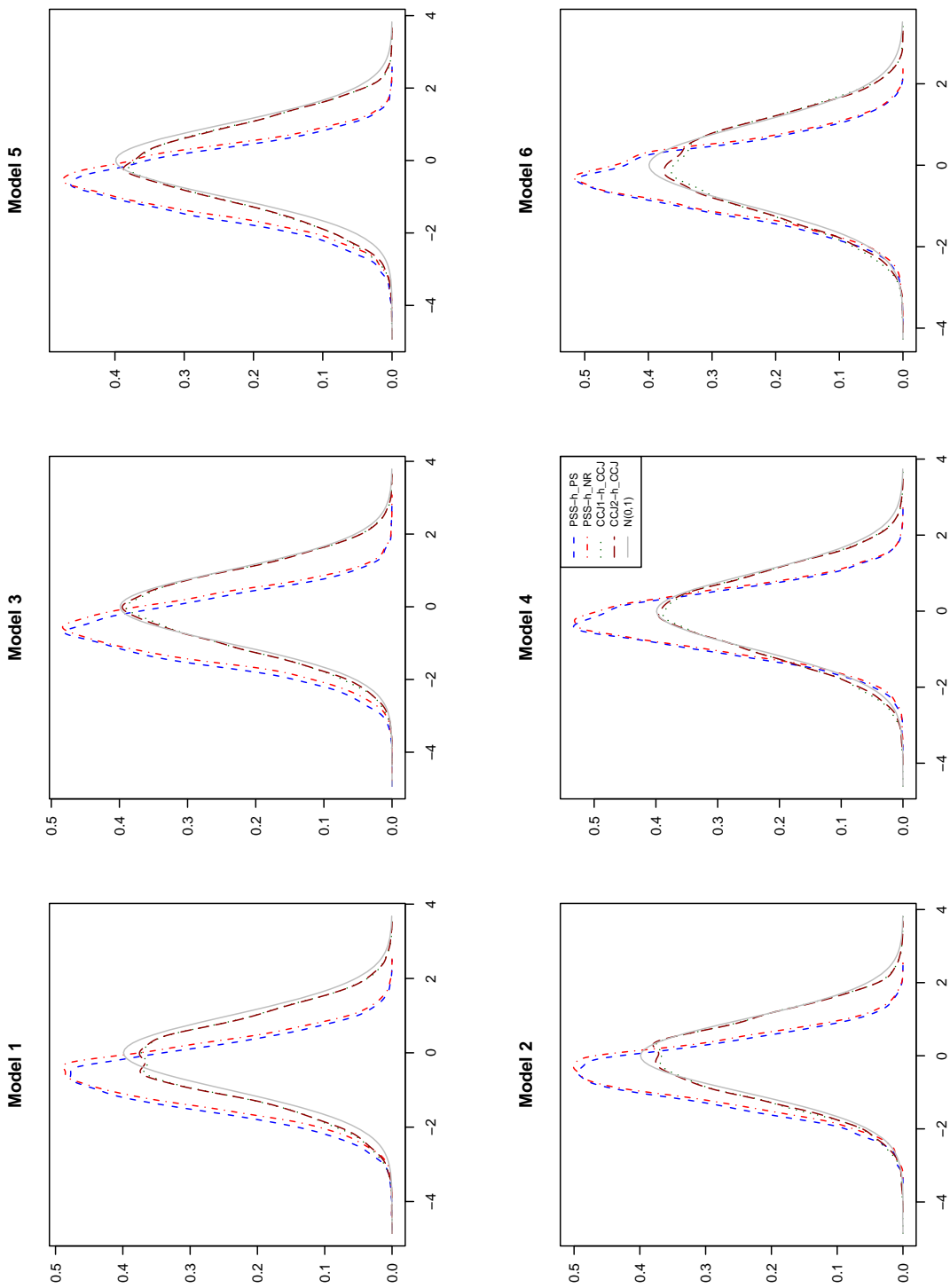


Figure 21: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4, P = 2, n = 700$

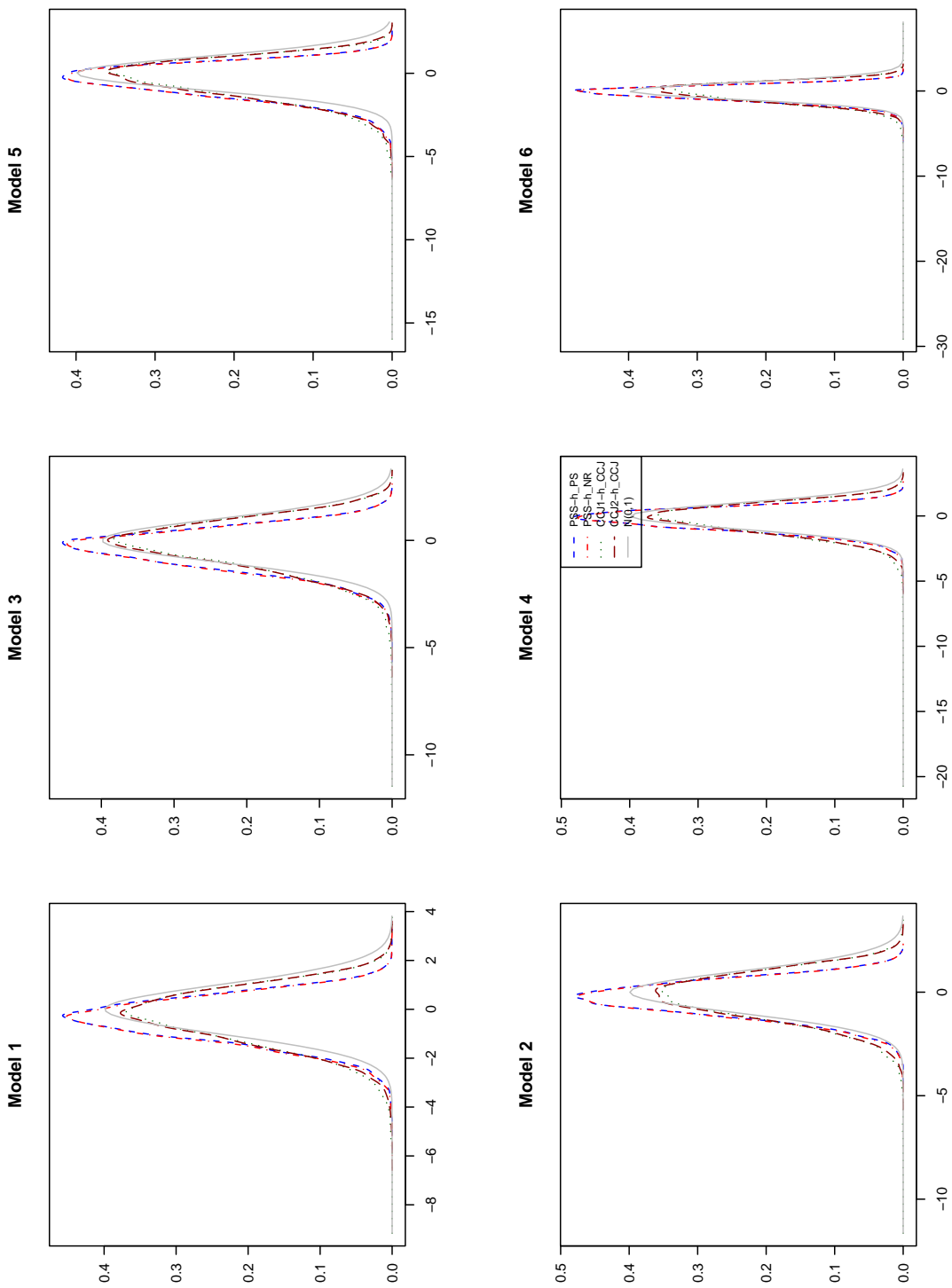


Figure 22: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 4$ ,  $n = 100$



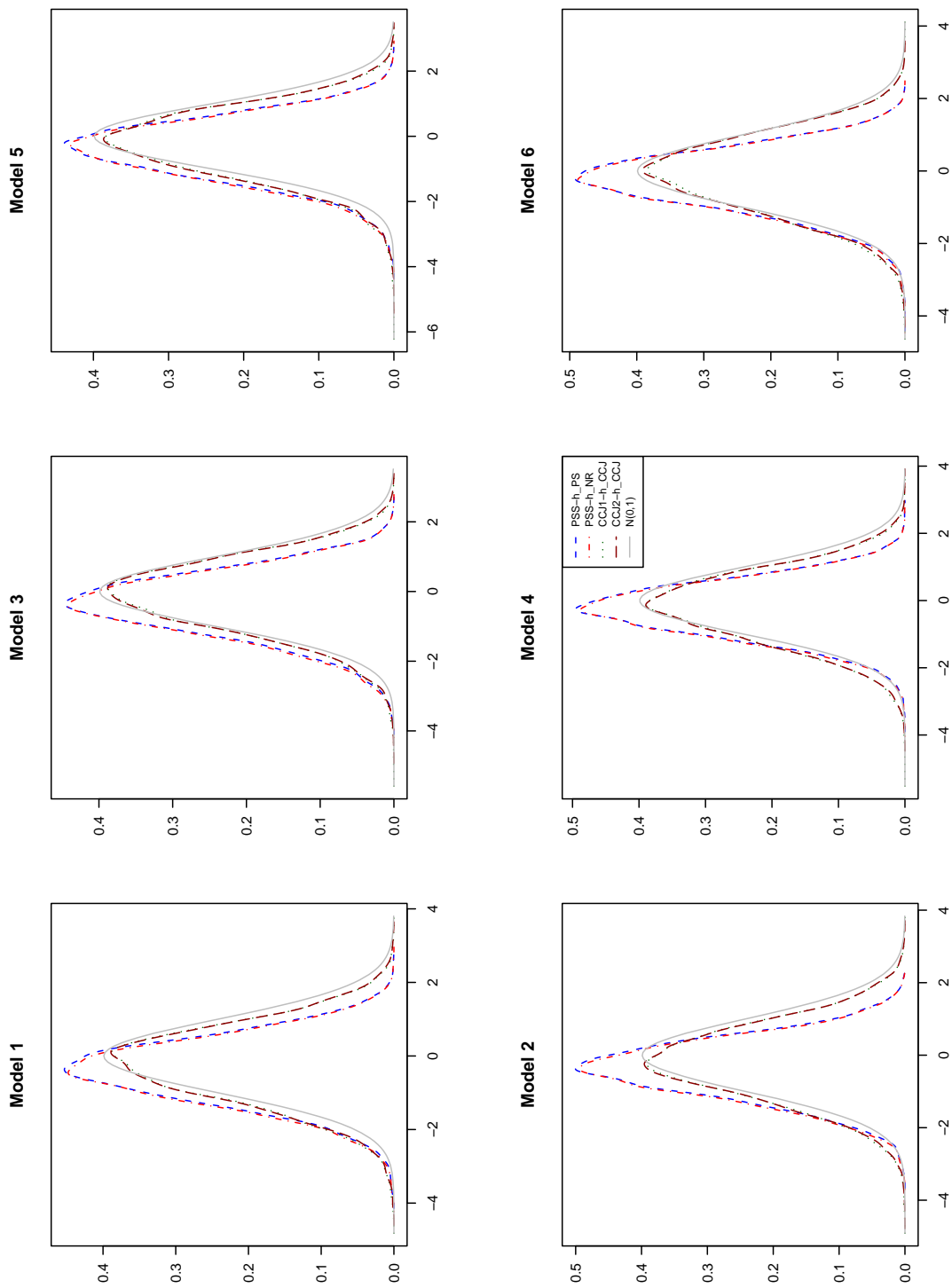


Figure 23: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 4$ ,  $n = 400$

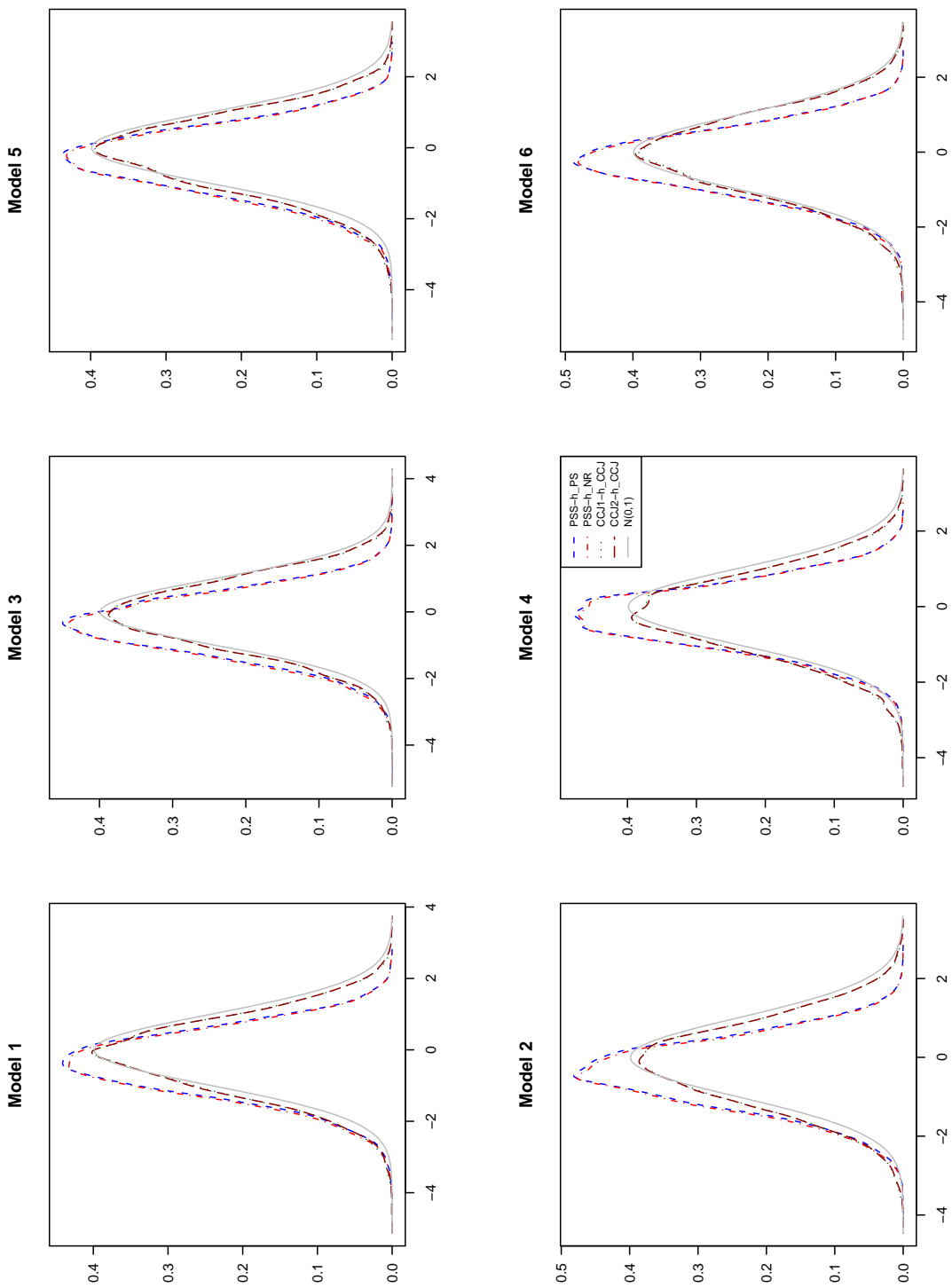


Figure 24: Empirical Coverage Rates for 95% Confidence Intervals:  $d = 4$ ,  $P = 4$ ,  $n = 700$

Table 1: Empirical Coverage Rates of 95% Confidence Intervals with Population Bandwidth:  $d = 2, n = 100$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.345	0.928	0.854	0.851	0.367	0.929	0.858	0.853	0.365	0.906	0.844	0.842
	$h_{NR}^*$	0.345	0.928	0.854	0.851	0.367	0.929	0.858	0.853	0.365	0.906	0.844	0.842
	$h_{CCJ}^*$	0.192	0.991	0.928	0.943	0.175	0.994	0.933	0.949	0.199	0.988	0.914	0.934
$P = 4$	$h_{PS}^*$	0.592	0.949	0.909	0.901	0.607	0.951	0.907	0.899	0.614	0.933	0.900	0.892
	$h_{NR}^*$	0.627	0.936	0.898	0.885	0.643	0.939	0.898	0.884	0.650	0.924	0.892	0.880
	$h_{CCJ}^*$	0.442	0.979	0.930	0.936	0.439	0.984	0.932	0.938	0.450	0.971	0.919	0.926
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.227	0.972	0.910	0.911	0.243	0.977	0.916	0.918	0.279	0.957	0.895	0.900
	$h_{NR}^*$	0.227	0.972	0.910	0.911	0.243	0.977	0.916	0.918	0.279	0.957	0.895	0.900
	$h_{CCJ}^*$	0.128	0.992	0.935	0.946	0.148	0.993	0.932	0.946	0.119	0.995	0.930	0.949
$P = 4$	$h_{PS}^*$	0.409	0.966	0.912	0.905	0.426	0.971	0.926	0.918	0.461	0.960	0.918	0.918
	$h_{NR}^*$	0.433	0.955	0.903	0.893	0.451	0.964	0.922	0.911	0.489	0.954	0.914	0.911
	$h_{CCJ}^*$	0.336	0.982	0.929	0.933	0.361	0.983	0.932	0.933	0.290	0.988	0.925	0.944

Note: Column BW reports population bandwidths.

Table 2: Empirical Coverage Rates of 95% Confidence Intervals with Population Bandwidth:  $d = 2, n = 400$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.244	0.931	0.878	0.876	0.260	0.939	0.887	0.881	0.258	0.929	0.885	0.880
	$h_{NR}^*$	0.244	0.931	0.878	0.876	0.260	0.939	0.887	0.881	0.258	0.929	0.885	0.880
	$h_{CCJ}^*$	0.121	0.994	0.948	0.952	0.110	0.995	0.947	0.954	0.125	0.993	0.947	0.951
$P = 4$	$h_{PS}^*$	0.470	0.949	0.926	0.920	0.483	0.951	0.930	0.921	0.488	0.941	0.925	0.918
	$h_{NR}^*$	0.498	0.940	0.920	0.912	0.512	0.943	0.925	0.912	0.517	0.935	0.918	0.910
	$h_{CCJ}^*$	0.335	0.978	0.942	0.943	0.333	0.981	0.945	0.945	0.342	0.975	0.940	0.941
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.161	0.970	0.921	0.916	0.172	0.978	0.935	0.931	0.197	0.968	0.920	0.919
	$h_{NR}^*$	0.161	0.970	0.921	0.916	0.172	0.978	0.935	0.931	0.197	0.968	0.920	0.919
	$h_{CCJ}^*$	0.081	0.994	0.944	0.946	0.093	0.993	0.947	0.949	0.074	0.995	0.946	0.950
$P = 4$	$h_{PS}^*$	0.325	0.951	0.917	0.907	0.338	0.964	0.938	0.927	0.366	0.962	0.936	0.931
	$h_{NR}^*$	0.344	0.940	0.909	0.897	0.358	0.958	0.933	0.922	0.388	0.956	0.931	0.926
	$h_{CCJ}^*$	0.254	0.977	0.940	0.939	0.273	0.982	0.945	0.943	0.220	0.990	0.945	0.949

Note: Column BW reports population bandwidths.

Table 3: Empirical Coverage Rates of 95% Confidence Intervals with Population Bandwidth:  $d = 2, n = 700$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.212	0.940	0.892	0.889	0.226	0.942	0.895	0.889	0.224	0.936	0.900	0.895
	$h_{NR}^*$	0.212	0.940	0.892	0.889	0.226	0.942	0.895	0.889	0.224	0.936	0.900	0.895
	$h_{CCJ}^*$	0.100	0.993	0.949	0.952	0.091	0.994	0.951	0.952	0.104	0.993	0.947	0.949
$P = 4$	$h_{PS}^*$	0.428	0.951	0.930	0.925	0.438	0.950	0.934	0.927	0.444	0.948	0.933	0.927
	$h_{NR}^*$	0.453	0.942	0.924	0.916	0.464	0.944	0.930	0.920	0.471	0.941	0.928	0.921
	$h_{CCJ}^*$	0.299	0.978	0.946	0.946	0.298	0.978	0.943	0.942	0.306	0.976	0.944	0.945
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.140	0.970	0.919	0.914	0.150	0.977	0.934	0.932	0.172	0.969	0.927	0.926
	$h_{NR}^*$	0.140	0.970	0.919	0.914	0.150	0.977	0.934	0.932	0.172	0.969	0.927	0.926
	$h_{CCJ}^*$	0.067	0.993	0.946	0.947	0.077	0.994	0.951	0.954	0.062	0.994	0.951	0.952
$P = 4$	$h_{PS}^*$	0.296	0.947	0.916	0.909	0.307	0.964	0.937	0.931	0.333	0.962	0.936	0.934
	$h_{NR}^*$	0.313	0.937	0.907	0.896	0.326	0.956	0.935	0.926	0.353	0.956	0.934	0.929
	$h_{CCJ}^*$	0.227	0.976	0.937	0.936	0.245	0.980	0.947	0.944	0.195	0.990	0.950	0.952

Note: Column BW reports population bandwidths.

Table 4: Empirical Coverage Rates of 95% Confidence Intervals with Population Bandwidth:  $d = 4, n = 100$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.410	0.900	0.788	0.796	0.440	0.888	0.783	0.788	0.431	0.876	0.764	0.773
	$h_{NR}^*$	0.393	0.921	0.815	0.828	0.422	0.912	0.811	0.817	0.414	0.898	0.790	0.804
	$h_{CCJ}^*$	0.300	0.977	0.903	0.922	0.252	0.976	0.898	0.939	0.312	0.968	0.882	0.908
$P = 4$	$h_{PS}^*$	0.679	0.951	0.878	0.882	0.705	0.942	0.875	0.875	0.703	0.928	0.858	0.863
	$h_{NR}^*$	0.693	0.946	0.873	0.875	0.719	0.935	0.867	0.864	0.717	0.919	0.850	0.853
	$h_{CCJ}^*$	0.575	0.977	0.913	0.925	0.528	0.983	0.913	0.931	0.589	0.966	0.896	0.912

Note: Column BW reports population bandwidths.

Table 5: Empirical Coverage Rates of 95% Confidence Intervals with Population Bandwidth:  $d = 4, n = 400$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.311	0.926	0.820	0.820	0.333	0.920	0.826	0.822	0.327	0.910	0.811	0.813
	$h_{NR}^*$	0.298	0.945	0.847	0.853	0.319	0.940	0.851	0.852	0.314	0.932	0.838	0.846
	$h_{CCJ}^*$	0.213	0.992	0.940	0.946	0.174	0.995	0.948	0.952	0.220	0.990	0.929	0.936
$P = 4$	$h_{PS}^*$	0.556	0.955	0.905	0.902	0.579	0.947	0.899	0.892	0.578	0.946	0.899	0.895
	$h_{NR}^*$	0.567	0.949	0.897	0.894	0.591	0.941	0.894	0.884	0.590	0.940	0.892	0.888
	$h_{CCJ}^*$	0.456	0.984	0.936	0.939	0.422	0.990	0.940	0.943	0.468	0.980	0.934	0.939

Note: Column BW reports population bandwidths.

Table 6: Empirical Coverage Rates of 95% Confidence Intervals with Population Bandwidth:  $d = 4, n = 700$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.278	0.932	0.830	0.832	0.298	0.926	0.831	0.826	0.292	0.922	0.831	0.832
	$h_{NR}^*$	0.267	0.950	0.860	0.862	0.286	0.945	0.859	0.859	0.281	0.943	0.858	0.861
	$h_{CCJ}^*$	0.185	0.992	0.941	0.944	0.155	0.996	0.948	0.951	0.192	0.992	0.938	0.944
$P = 4$	$h_{PS}^*$	0.514	0.955	0.914	0.911	0.535	0.952	0.912	0.904	0.532	0.949	0.910	0.907
	$h_{NR}^*$	0.524	0.950	0.908	0.904	0.546	0.948	0.906	0.896	0.543	0.943	0.906	0.901
	$h_{CCJ}^*$	0.416	0.986	0.938	0.941	0.387	0.990	0.944	0.946	0.427	0.982	0.937	0.941

Note: Column BW reports population bandwidths.

Table 7: Empirical Average Length of 95% Confidence Intervals with Population Bandwidth:  $d = 2, n = 100$ .

		Model 1				Model 3				Model 5			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.009	0.068	0.057	0.055	0.003	0.024	0.021	0.020	0.005	0.040	0.035	0.033
	$h_{NR}^*$	0.009	0.068	0.057	0.055	0.003	0.024	0.021	0.020	0.005	0.040	0.035	0.033
	$h_{CCJ}^*$	0.003	0.175	0.127	0.128	0.001	0.084	0.059	0.061	0.002	0.101	0.074	0.075
$P = 4$	$h_{PS}^*$	0.373	6.400	5.597	5.341	0.149	2.388	2.094	1.962	0.214	3.969	3.547	3.361
	$h_{NR}^*$	0.453	6.016	5.344	5.038	0.177	2.234	1.990	1.839	0.259	3.751	3.401	3.183
	$h_{CCJ}^*$	0.140	9.218	7.349	7.337	0.055	3.667	2.896	2.877	0.073	5.672	4.594	4.585

Note: Column BIAS reports absolute difference between average of  $\hat{\theta}_n$  (across simulations) and  $\theta_0$ . All figures times 100.

Table 8: Empirical Average Length of 95% Confidence Intervals with Population Bandwidth:  $d = 2, n = 400$ .

		Model 1				Model 3				Model 5			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.005	0.036	0.031	0.030	0.002	0.013	0.011	0.011	0.003	0.022	0.019	0.019
	$h_{NR}^*$	0.005	0.036	0.031	0.030	0.002	0.013	0.011	0.011	0.003	0.022	0.019	0.019
	$h_{CCJ}^*$	0.002	0.110	0.080	0.080	0.000	0.053	0.038	0.038	0.001	0.064	0.047	0.047
$P = 4$	$h_{PS}^*$	0.183	3.096	2.842	2.755	0.050	1.184	1.091	1.043	0.090	1.981	1.849	1.782
	$h_{NR}^*$	0.221	2.971	2.762	2.657	0.065	1.133	1.057	1.001	0.112	1.909	1.802	1.723
	$h_{CCJ}^*$	0.070	4.302	3.566	3.556	0.002	1.720	1.417	1.409	0.021	2.696	2.279	2.268

Note: Column BIAS reports absolute difference between average of  $\hat{\theta}_n$  (across simulations) and  $\theta_0$ . All figures times 100.

Table 9: Empirical Average Length of 95% Confidence Intervals with Population Bandwidth:  $d = 2, n = 700$ .

		Model 1				Model 3				Model 5			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.003	0.028	0.024	0.023	0.001	0.010	0.009	0.009	0.002	0.017	0.015	0.015
	$h_{NR}^*$	0.003	0.028	0.024	0.023	0.001	0.010	0.009	0.009	0.002	0.017	0.015	0.015
	$h_{CCJ}^*$	0.001	0.091	0.065	0.066	0.000	0.044	0.032	0.032	0.000	0.053	0.038	0.039
$P = 4$	$h_{PS}^*$	0.124	2.316	2.156	2.103	0.046	0.894	0.835	0.805	0.071	1.490	1.408	1.367
	$h_{NR}^*$	0.152	2.238	2.107	2.042	0.056	0.862	0.814	0.778	0.087	1.446	1.379	1.330
	$h_{CCJ}^*$	0.036	3.163	2.656	2.652	0.010	1.270	1.059	1.055	0.019	1.986	1.700	1.695

Note: Column BIAS reports absolute difference between average of  $\hat{\theta}_n$  (across simulations) and  $\theta_0$ . All figures times 100.

Table 10: Empirical Average Length of 95% Confidence Intervals with Population Bandwidth:  $d = 4, n = 100$ .

		Model 1				Model 3				Model 5			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.001	0.008	0.006	0.006	0.000	0.002	0.002	0.002	0.001	0.005	0.004	0.004
	$h_{NR}^*$	0.001	0.009	0.007	0.007	0.000	0.003	0.002	0.002	0.001	0.005	0.004	0.004
	$h_{CCJ}^*$	0.001	0.018	0.013	0.014	0.000	0.010	0.007	0.008	0.000	0.011	0.008	0.008
$P = 4$	$h_{PS}^*$	0.064	0.807	0.648	0.633	0.019	0.245	0.199	0.190	0.037	0.476	0.389	0.376
	$h_{NR}^*$	0.068	0.772	0.625	0.608	0.020	0.234	0.192	0.182	0.039	0.455	0.376	0.362
	$h_{CCJ}^*$	0.039	1.190	0.899	0.903	0.007	0.506	0.370	0.373	0.021	0.712	0.543	0.544

		Model 2				Model 4				Model 6			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.002	0.017	0.013	0.012	0.000	0.006	0.005	0.005	0.001	0.008	0.006	0.006
	$h_{NR}^*$	0.002	0.019	0.014	0.014	0.000	0.007	0.005	0.005	0.001	0.009	0.007	0.007
	$h_{CCJ}^*$	0.001	0.058	0.041	0.042	0.000	0.016	0.011	0.012	0.000	0.035	0.025	0.026
$P = 4$	$h_{PS}^*$	0.107	1.775	1.337	1.326	0.027	0.599	0.450	0.445	0.034	0.855	0.646	0.649
	$h_{NR}^*$	0.112	1.683	1.276	1.262	0.028	0.567	0.429	0.423	0.036	0.813	0.619	0.619
	$h_{CCJ}^*$	0.067	2.990	2.157	2.180	0.022	0.843	0.612	0.617	0.013	2.139	1.511	1.560

Note: Column BIAS reports absolute difference between average of  $\hat{\theta}_n$  (across simulations) and  $\theta_0$ . All figures times 100.

Table 11: Empirical Average Length of 95% Confidence Intervals with Population Bandwidth:  $d = 4, n = 400$ .

		Model 1				Model 3				Model 5			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.001	0.005	0.004	0.004	0.000	0.001	0.001	0.001	0.000	0.003	0.002	0.002
	$h_{NR}^*$	0.001	0.005	0.004	0.004	0.000	0.002	0.001	0.001	0.000	0.003	0.002	0.002
	$h_{CCJ}^*$	0.000	0.013	0.009	0.010	0.000	0.009	0.006	0.006	0.000	0.008	0.005	0.006
$P = 4$	$h_{PS}^*$	0.034	0.399	0.334	0.327	0.010	0.126	0.107	0.103	0.020	0.240	0.206	0.200
	$h_{NR}^*$	0.037	0.384	0.325	0.317	0.011	0.121	0.104	0.099	0.021	0.232	0.201	0.194
	$h_{CCJ}^*$	0.018	0.611	0.470	0.471	0.004	0.260	0.194	0.194	0.010	0.368	0.287	0.286

		Model 2				Model 4				Model 6			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.001	0.010	0.007	0.007	0.000	0.004	0.003	0.003	0.000	0.005	0.004	0.004
	$h_{NR}^*$	0.001	0.011	0.008	0.008	0.000	0.004	0.003	0.003	0.000	0.006	0.004	0.004
	$h_{CCJ}^*$	0.001	0.044	0.031	0.031	0.000	0.013	0.009	0.009	0.000	0.030	0.021	0.022
$P = 4$	$h_{PS}^*$	0.072	0.855	0.667	0.657	0.017	0.296	0.231	0.227	0.022	0.416	0.326	0.323
	$h_{NR}^*$	0.076	0.814	0.640	0.629	0.018	0.282	0.222	0.217	0.023	0.396	0.314	0.310
	$h_{CCJ}^*$	0.045	1.561	1.141	1.142	0.013	0.449	0.333	0.331	0.008	1.174	0.839	0.847

Note: Column BIAS reports absolute difference between average of  $\hat{\theta}_n$  (across simulations) and  $\theta_0$ . All figures times 100.

Table 12: Empirical Average Length of 95% Confidence Intervals with Population Bandwidth:  $d = 4, n = 700$ .

		Model 1				Model 3				Model 5			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.001	0.004	0.003	0.003	0.000	0.001	0.001	0.001	0.000	0.002	0.002	0.002
	$h_{NR}^*$	0.001	0.004	0.003	0.003	0.000	0.001	0.001	0.001	0.000	0.002	0.002	0.002
	$h_{CCJ}^*$	0.000	0.011	0.008	0.008	0.000	0.007	0.005	0.005	0.000	0.007	0.005	0.005
$P = 4$	$h_{PS}^*$	0.025	0.298	0.253	0.249	0.008	0.095	0.082	0.079	0.014	0.182	0.158	0.154
	$h_{NR}^*$	0.027	0.288	0.247	0.242	0.009	0.092	0.080	0.077	0.015	0.176	0.154	0.150
	$h_{CCJ}^*$	0.012	0.463	0.358	0.358	0.003	0.195	0.147	0.146	0.006	0.279	0.219	0.219

		Model 2				Model 4				Model 6			
		BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2	BIAS	PSS	CCJ1	CCJ2
$P = 2$	$h_{PS}^*$	0.001	0.008	0.006	0.006	0.000	0.003	0.002	0.002	0.000	0.004	0.003	0.003
	$h_{NR}^*$	0.001	0.009	0.007	0.007	0.000	0.003	0.003	0.003	0.000	0.004	0.003	0.003
	$h_{CCJ}^*$	0.000	0.038	0.027	0.027	0.000	0.011	0.008	0.008	0.000	0.025	0.018	0.018
$P = 4$	$h_{PS}^*$	0.059	0.634	0.502	0.493	0.013	0.221	0.174	0.171	0.017	0.307	0.244	0.242
	$h_{NR}^*$	0.062	0.605	0.482	0.473	0.014	0.210	0.168	0.164	0.018	0.294	0.235	0.233
	$h_{CCJ}^*$	0.032	1.187	0.872	0.870	0.009	0.344	0.256	0.254	0.006	0.860	0.617	0.621

Note: Column BIAS reports absolute difference between average of  $\hat{\theta}_n$  (across simulations) and  $\theta_0$ . All figures times 100.

Table 13: Empirical Coverage Rates of 95% Confidence Intervals with Estimated Bandwidth:  $d = 2, n = 100$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.327	0.884	0.808	0.805	0.330	0.897	0.815	0.810	0.331	0.888	0.823	0.822
	$\hat{h}_{NR}$	0.327	0.884	0.808	0.805	0.330	0.897	0.815	0.810	0.331	0.888	0.823	0.822
	$\hat{h}_{CCJ}$	0.182	0.971	0.916	0.927	0.198	0.972	0.906	0.924	0.194	0.968	0.900	0.919
$P = 4$	$\hat{h}_{PS}$	0.373	0.980	0.905	0.917	0.374	0.982	0.903	0.921	0.374	0.970	0.897	0.911
	$\hat{h}_{NR}$	0.395	0.976	0.903	0.912	0.397	0.978	0.902	0.914	0.397	0.966	0.896	0.907
	$\hat{h}_{CCJ}$	0.271	0.992	0.928	0.948	0.284	0.991	0.918	0.940	0.280	0.988	0.916	0.940
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.278	0.864	0.789	0.775	0.282	0.908	0.835	0.820	0.284	0.919	0.838	0.840
	$\hat{h}_{NR}$	0.278	0.864	0.789	0.775	0.282	0.908	0.835	0.820	0.284	0.919	0.838	0.840
	$\hat{h}_{CCJ}$	0.167	0.961	0.898	0.905	0.182	0.968	0.907	0.915	0.170	0.976	0.919	0.934
$P = 4$	$\hat{h}_{PS}$	0.322	0.970	0.898	0.903	0.322	0.980	0.910	0.920	0.323	0.980	0.900	0.914
	$\hat{h}_{NR}$	0.341	0.962	0.892	0.894	0.341	0.977	0.907	0.913	0.342	0.976	0.898	0.910
	$\hat{h}_{CCJ}$	0.246	0.987	0.922	0.934	0.253	0.990	0.922	0.939	0.246	0.991	0.921	0.943

Note: Column BW reports sample mean of estimated bandwidths.

Table 14: Empirical Coverage Rates of 95% Confidence Intervals with Estimated Bandwidth:  $d = 2, n = 400$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.248	0.870	0.817	0.809	0.255	0.883	0.819	0.809	0.252	0.887	0.833	0.823
	$\hat{h}_{NR}$	0.248	0.870	0.817	0.809	0.255	0.883	0.819	0.809	0.252	0.887	0.833	0.823
	$\hat{h}_{CCJ}$	0.113	0.980	0.937	0.940	0.132	0.976	0.932	0.932	0.120	0.981	0.938	0.941
$P = 4$	$\hat{h}_{PS}$	0.290	0.978	0.921	0.924	0.290	0.980	0.922	0.923	0.290	0.979	0.923	0.926
	$\hat{h}_{NR}$	0.308	0.975	0.921	0.922	0.307	0.977	0.921	0.921	0.308	0.975	0.921	0.922
	$\hat{h}_{CCJ}$	0.187	0.993	0.949	0.953	0.198	0.994	0.948	0.954	0.192	0.995	0.949	0.954
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.201	0.858	0.796	0.780	0.208	0.903	0.851	0.838	0.212	0.920	0.860	0.854
	$\hat{h}_{NR}$	0.201	0.858	0.796	0.780	0.208	0.903	0.851	0.838	0.212	0.920	0.860	0.854
	$\hat{h}_{CCJ}$	0.104	0.972	0.916	0.919	0.119	0.973	0.929	0.930	0.105	0.986	0.943	0.946
$P = 4$	$\hat{h}_{PS}$	0.239	0.975	0.912	0.911	0.241	0.981	0.925	0.925	0.241	0.986	0.922	0.925
	$\hat{h}_{NR}$	0.254	0.967	0.908	0.906	0.255	0.976	0.925	0.921	0.256	0.981	0.919	0.921
	$\hat{h}_{CCJ}$	0.166	0.991	0.942	0.945	0.175	0.993	0.943	0.948	0.164	0.995	0.951	0.958

Note: Column BW reports sample mean of estimated bandwidths.

Table 15: Empirical Coverage Rates of 95% Confidence Intervals with Estimated Bandwidth:  $d = 2, n = 700$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.220	0.871	0.820	0.811	0.225	0.886	0.834	0.825	0.224	0.892	0.846	0.838
	$\hat{h}_{NR}$	0.220	0.871	0.820	0.811	0.225	0.886	0.834	0.825	0.224	0.892	0.846	0.838
	$\hat{h}_{CCJ}$	0.095	0.984	0.939	0.941	0.108	0.982	0.944	0.946	0.098	0.985	0.947	0.949
$P = 4$	$\hat{h}_{PS}$	0.261	0.980	0.923	0.924	0.261	0.981	0.931	0.930	0.260	0.978	0.925	0.926
	$\hat{h}_{NR}$	0.276	0.976	0.923	0.922	0.277	0.976	0.929	0.928	0.276	0.974	0.925	0.926
	$\hat{h}_{CCJ}$	0.161	0.993	0.952	0.956	0.171	0.994	0.953	0.955	0.164	0.994	0.952	0.954
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.178	0.849	0.796	0.783	0.182	0.902	0.849	0.836	0.185	0.922	0.869	0.862
	$\hat{h}_{NR}$	0.178	0.849	0.796	0.783	0.182	0.902	0.849	0.836	0.185	0.922	0.869	0.862
	$\hat{h}_{CCJ}$	0.088	0.973	0.926	0.925	0.100	0.978	0.938	0.936	0.085	0.989	0.951	0.952
$P = 4$	$\hat{h}_{PS}$	0.213	0.970	0.918	0.916	0.212	0.983	0.928	0.928	0.212	0.984	0.927	0.928
	$\hat{h}_{NR}$	0.225	0.963	0.915	0.913	0.225	0.980	0.928	0.924	0.225	0.982	0.926	0.927
	$\hat{h}_{CCJ}$	0.143	0.991	0.948	0.949	0.151	0.991	0.947	0.950	0.138	0.995	0.950	0.954

Note: Column BW reports sample mean of estimated bandwidths.

Table 16: Empirical Coverage Rates of 95% Confidence Intervals with Estimated Bandwidth:  $d = 4, n = 100$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.297	0.948	0.856	0.882	0.297	0.950	0.865	0.903	0.297	0.944	0.850	0.884
	$\hat{h}_{NR}$	0.286	0.957	0.869	0.898	0.285	0.955	0.876	0.917	0.285	0.950	0.859	0.898
	$\hat{h}_{CCJ}$	0.218	0.971	0.912	0.950	0.222	0.943	0.879	0.944	0.224	0.955	0.887	0.935
$P = 4$	$\hat{h}_{PS}$	0.374	0.987	0.908	0.936	0.374	0.979	0.902	0.943	0.373	0.980	0.898	0.934
	$\hat{h}_{NR}$	0.382	0.986	0.907	0.933	0.381	0.980	0.902	0.938	0.381	0.980	0.896	0.932
	$\hat{h}_{CCJ}$	0.324	0.989	0.925	0.957	0.332	0.974	0.905	0.958	0.330	0.980	0.908	0.953
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.256	0.953	0.866	0.891	0.256	0.953	0.876	0.918	0.257	0.958	0.875	0.914
	$\hat{h}_{NR}$	0.246	0.959	0.877	0.904	0.246	0.955	0.881	0.927	0.247	0.960	0.883	0.923
	$\hat{h}_{CCJ}$	0.192	0.968	0.908	0.951	0.197	0.932	0.868	0.939	0.201	0.957	0.889	0.945
$P = 4$	$\hat{h}_{PS}$	0.323	0.987	0.911	0.937	0.322	0.975	0.903	0.950	0.322	0.980	0.906	0.944
	$\hat{h}_{NR}$	0.330	0.986	0.909	0.933	0.328	0.976	0.903	0.947	0.329	0.981	0.905	0.941
	$\hat{h}_{CCJ}$	0.286	0.987	0.921	0.956	0.293	0.971	0.902	0.957	0.293	0.978	0.906	0.956

Note: Column BW reports sample mean of estimated bandwidths.

Table 17: Empirical Coverage Rates of 95% Confidence Intervals with Estimated Bandwidth:  $d = 4, n = 400$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.230	0.960	0.886	0.893	0.230	0.970	0.903	0.909	0.230	0.967	0.893	0.904
	$\hat{h}_{NR}$	0.221	0.967	0.900	0.906	0.221	0.975	0.914	0.921	0.220	0.973	0.905	0.916
	$\hat{h}_{CCJ}$	0.151	0.988	0.955	0.962	0.152	0.989	0.957	0.974	0.155	0.989	0.953	0.966
$P = 4$	$\hat{h}_{PS}$	0.291	0.994	0.931	0.938	0.291	0.994	0.937	0.944	0.290	0.995	0.932	0.941
	$\hat{h}_{NR}$	0.297	0.993	0.929	0.936	0.297	0.994	0.934	0.942	0.296	0.994	0.930	0.939
	$\hat{h}_{CCJ}$	0.233	0.996	0.953	0.961	0.237	0.998	0.955	0.966	0.239	0.996	0.953	0.965
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.190	0.969	0.904	0.911	0.189	0.980	0.917	0.926	0.191	0.980	0.916	0.927
	$\hat{h}_{NR}$	0.182	0.974	0.913	0.922	0.181	0.984	0.926	0.937	0.183	0.984	0.925	0.938
	$\hat{h}_{CCJ}$	0.128	0.990	0.957	0.969	0.130	0.988	0.959	0.981	0.135	0.989	0.957	0.976
$P = 4$	$\hat{h}_{PS}$	0.240	0.994	0.935	0.942	0.240	0.996	0.939	0.946	0.240	0.995	0.934	0.945
	$\hat{h}_{NR}$	0.245	0.993	0.934	0.939	0.245	0.996	0.936	0.943	0.245	0.995	0.931	0.942
	$\hat{h}_{CCJ}$	0.199	0.997	0.954	0.964	0.201	0.997	0.959	0.971	0.203	0.998	0.953	0.969

Note: Column BW reports sample mean of estimated bandwidths.

Table 18: Empirical Coverage Rates of 95% Confidence Intervals with Estimated Bandwidth:  $d = 4, n = 700$ .

		Model 1				Model 3				Model 5			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.205	0.963	0.892	0.896	0.207	0.971	0.907	0.910	0.207	0.971	0.905	0.911
	$\hat{h}_{NR}$	0.197	0.970	0.905	0.910	0.199	0.977	0.916	0.920	0.199	0.976	0.916	0.921
	$\hat{h}_{CCJ}$	0.130	0.992	0.956	0.962	0.132	0.992	0.961	0.970	0.135	0.991	0.956	0.968
$P = 4$	$\hat{h}_{PS}$	0.260	0.993	0.931	0.936	0.262	0.994	0.936	0.941	0.262	0.994	0.930	0.938
	$\hat{h}_{NR}$	0.266	0.992	0.928	0.934	0.268	0.994	0.934	0.939	0.267	0.994	0.928	0.937
	$\hat{h}_{CCJ}$	0.203	0.996	0.953	0.958	0.208	0.998	0.956	0.961	0.209	0.996	0.956	0.961
		Model 2				Model 4				Model 6			
		BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2	BW	PSS	CCJ1	CCJ2
$P = 2$	$\hat{h}_{PS}$	0.169	0.970	0.908	0.910	0.168	0.982	0.925	0.930	0.168	0.982	0.925	0.930
	$\hat{h}_{NR}$	0.162	0.975	0.917	0.922	0.161	0.986	0.932	0.936	0.161	0.985	0.931	0.938
	$\hat{h}_{CCJ}$	0.110	0.992	0.964	0.967	0.111	0.992	0.968	0.979	0.114	0.993	0.963	0.973
$P = 4$	$\hat{h}_{PS}$	0.213	0.994	0.941	0.944	0.213	0.996	0.938	0.943	0.213	0.996	0.935	0.940
	$\hat{h}_{NR}$	0.217	0.994	0.938	0.940	0.217	0.995	0.935	0.940	0.217	0.995	0.933	0.937
	$\hat{h}_{CCJ}$	0.171	0.997	0.957	0.963	0.174	0.998	0.958	0.966	0.176	0.998	0.957	0.964

Note: Column BW reports sample mean of estimated bandwidths.